THRUST RESTRAINT DESIGN PROGRAM

USER'S GUIDE

SEPTEMBER 2013

AMERICAN Concrete Pressure Pipe ASSOCIATION

DURABLE
ECONOMICAL
ADAPTABLE
RELIABLE
SUSTAINABLE

THE THRUST RESTRAINT DESIGN PROGRAM (TRDP) was specifically developed by the American Concrete Pressure Pipe Association (ACPPA) and the consulting firm of Simpson, Gumpertz & Heger to aid in the design of restraint systems for buried concrete pressure pipelines using restrained joints. The interactive software provides users with an accurate, reliable and streamlined process for designing thrust restraint systems in accordance with the American Water Works Association's ***Concrete Pressure Pipe, Manual of Water Supply Practices, Third Edition (AWWA Manual M9)*** and all applicable standards for the design and manufacture of concrete pressure pipe.

System Requirements

The minimum computer configurations required to run the TRDP application are:

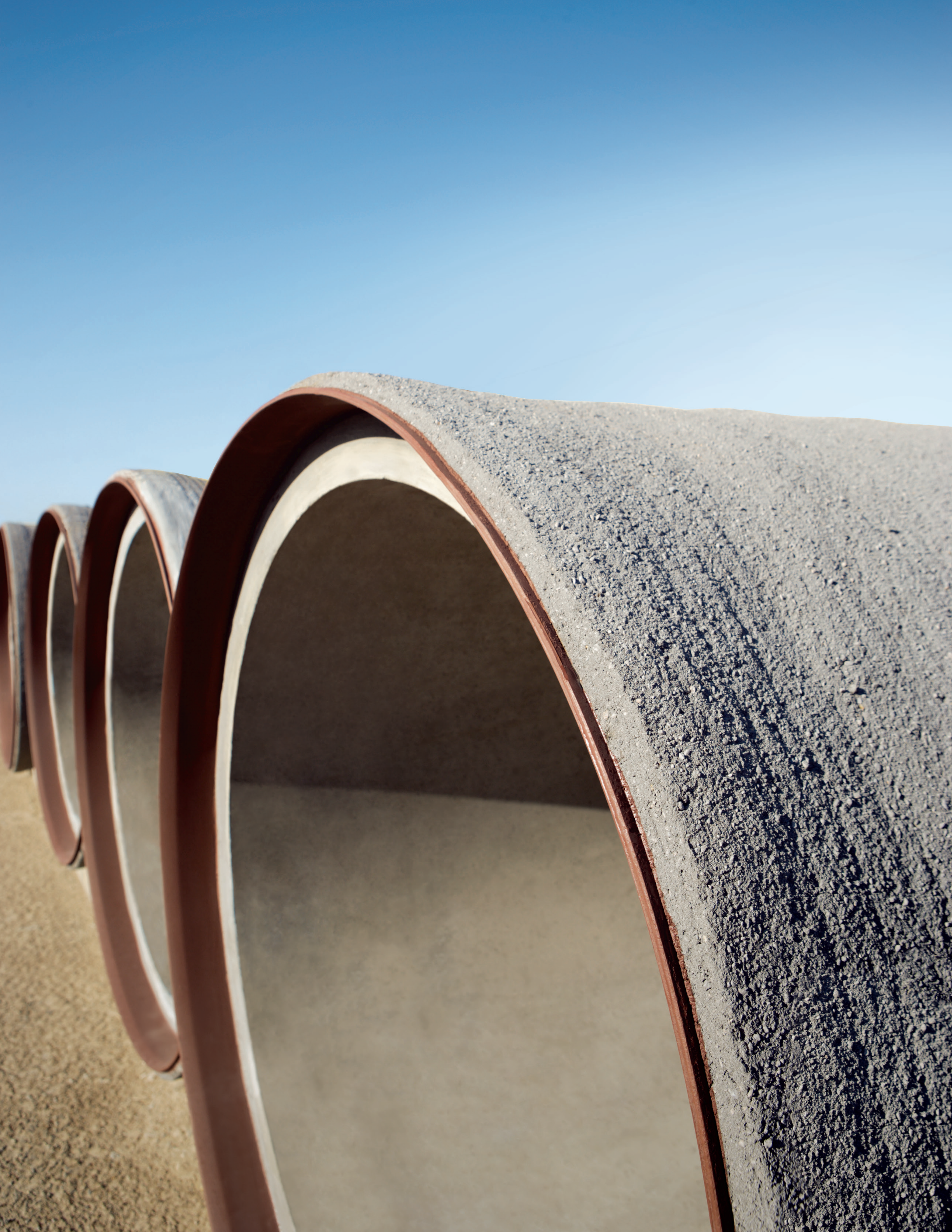
- Microsoft Windows 2000 (with Service Pack 4) or later;
- Microsoft Office 2003 or later;
- Microsoft.Net Framework Version 2.0 with Service Pack 1 or greater;
- If Microsoft.Net Framework Version 2.0 is not installed on the target computer, it will be installed automatically. If the software is already installed, the user will be asked to verify it is up-to-date.
- Internet Explorer 5.01 or later; and
- Access to a printer.

Disclaimer

TRDP is intended to be used by a licensed Professional Engineer (PE) experienced with the techniques for designing and/or restraining concrete pressure pipelines. Non-licensed engineers and technicians may use the software after appropriate training and under the direct supervision of a PE.

The software is exceptionally complex by its nature. The software user assumes sole responsibility for professional decisions made with the assistance of the software, and must always exercise the usual standard of care and independent professional engineering judgment in analyzing and interpreting conclusions reached with the assistance of the software.

ACPPA will not be liable in any manner for the inability of the software to be used on any specific or particular engineering project. In no event will ACPPA or its distributor be liable to the software user or any third party for any damages, including lost profits, lost savings, lost revenues, loss of data or any indirect, special, incidental or consequential damages arising out of the use of or the inability to use the software, even if ACPPA has been previously advised of the possibility of such damages.



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ACPPA Headquarters
3900 University Drive
Suite 110
Fairfax, VA 22030-2513

T 703.273.7227
F 703.273.7230

www.acppa.org

APPLICATIONS

JDF assumptions in red or highlighted throughout

Unbalanced thrust forces generated at directional changes in concrete pressure pipelines, such as elbow fittings or dead-end bulkheads, can cause separation of unrestrained bell-and-spigot type pipe joints. If frictional drag and passive soil resistance forces on the fitting are not adequate to resist this movement, mitigating measures must be employed to prevent joint separation. For buried concrete pressure pipelines, there are two (2) acceptable procedures:

1. Provide restrained joints which mobilize the resistance forces generated between the exterior of multiple pipes and the surrounding soil; or

2. Install a concrete thrust block or thrust collar.

TRDP presents the user with a convenient means of determining the number of pipe sections that must be restrained to the fitting in order to resist the unbalanced thrust forces. Specifically, TRDP addresses restraint requirements for Prestressed Concrete Embedded Cylinder Pipe (ECP, AWWA C301), Prestressed Concrete Lined Cylinder Pipe (LCP, AWWA C301) and Bar-Wrapped Concrete Cylinder Pipe (BWP, AWWA C303) buried in various soil types at horizontal elbows and terminations, such as bulkheads and tee fittings. Reinforced Concrete Cylinder Pipe (RCCP, AWWA C300) will be included in a later version of the software. The soil type is an input to the program, since passive soil resistance provides a portion of the resistance to the unbalanced thrust forces.

The design program calculates the unbalanced thrust forces (bending moment, axial and shear) at the fitting, determines the number of pipe joints that must be restrained to the fitting and calculates the minimum steel cylinder thicknesses needed to transfer the forces through each restrained pipe or fitting. Within these computations, the program takes into account whether the joints are restrained mechanically (such as with a harness clamp or snap ring) or via field-welding. Pipelines with field-welded restrained joints do not allow a small amount of movement under load, as do mechanically restrained joints. Therefore, the use of welded joints for restraint typically requires somewhat thicker steel cylinders and longer restrained lengths than mechanically restrained joints.

TRDP also addresses restraint requirements at vertical elbows (long side bottom) with the unbalanced thrust pushing down into the soil using the same design procedure as for horizontal elbows. The design program does not address vertical elbows (long side top) where the unbalanced thrust pushes up out of the soil. In this situation, other design procedures must be used.

Vertical Up (Lower) Bend =Horizontal Bend

DEFINITIONS/ASSUMPTIONS

Types of Restrained Joints

Pipe joints must be restrained so that thrust is transmitted across the joint. There are two (2) types of restrained joints:

1. **Welded Joints** – Bell-and-spigot type joints restrained by welding either externally or internally. The thickness of each joint ring must be sufficient to safely transfer the thrust force to the steel cylinder.

2. **Mechanically Restrained Joints** – Bell-and-spigot type joints mechanically restrained together (harnessed or tied) to achieve the desired restraint by transmitting longitudinal thrust across a specific number of joints over the required length. Mechanical restraint can be achieved by the harness arrangements detailed in AWWA Manual M9, Figure 9-25, "Details of Typical Harnessed Joints."

Assumptions

- US Customary Units (a metric unit version will be available in a later version)
- Steel cylinder not thicker than 0.50 inch
- Pipe not submerged (submerged pipe option will be available in a later version)
- Pipe is ECP (AWWA C301), LCP (AWWA C301) or BWP (AWWA C303). The option to use RCCP (AWWA C300) will be available in a later version of the software
- Backfill around restrained-joint pipes and fittings where soil stiffness and friction will be providing thrust resistance will comply with the soil properties selected for design purposes (Refer to Chapter 6, Item 5 for additional information)
- Excavation of soil along a restrained joint, pipe or fitting is not permitted while a pipeline is under pressure
- TRDP considers the resultant thrust at a bend based on forces due to internal pressure only (i.e., no hydrodynamic forces), since most waterlines operate at relatively low velocities
- Small angular deflections at joints in full-length standard or bevel pipe do not require restraint
- Bends greater than 5° should be restrained
- A conservative estimate of slack ($\Omega = {}^{1}/_{16}$ inch) is used for mechanically harnessed joints to calculate bending moments in the pipe segments
- The effects of temperature and Poisson's Ratio are ignored in accordance with AWWA Manual M9, Chapter 9

DEFINITIONS/ASSUMPTIONS *(CONTINUED)*

Nomenclature

ECP	Prestressed Concrete Embedded Cylinder Pipe, AWWA C301
LCP	Prestressed Concrete Lined Cylinder Pipe, AWWA C301
BWP	Bar-Wrapped Concrete Cylinder Pipe, AWWA C303
ID	Nominal pipe inside diameter, inch
t_c	Core thickness, inch
D_y	Cylinder outside diameter, inch
t_y	Steel cylinder thickness, inch
t_m	Mortar coating thickness, inch
W_p	Pipe weight, pounds/foot (lb/ft)
P_w	Working pressure, pounds/square inch (psi)
P_t	Transient pressure, pounds/square inch (psi)
P_{ft}	Field test pressure, pounds/square inch (psi)
f_y	Steel cylinder yield strength, pounds/square inch (psi)
f'_c	28-day concrete compressive strength, pounds/square inch (psi)
Soil Type	I, II, III, IV or V as described in *AWWA Manual M9*, Table 9-1
k	Soil stiffness, pounds/square inch (psi)
γ	Unit weight of soil, pounds/cubic foot (pcf)
μ	Coefficient of friction between pipe exterior and soil
ϕ	Soil angle of internal friction
Joint Dia	Diameter of critical sealing surface of joint, inch

DATA INPUT

ASSUMED THAT CALCS ARE FOR VERTICAL UPWARD BENDS AND HORIZONTAL BENDS SINCE M-9 EQUATION IS THE SAME. PRESIDENT OF ACPPA, RICHARD MUELLER, AGREES WITH THIS IF MAINTAINING SAME DEPTH OF COVER FOR A VERTICAL UPWARD BEND.

After accepting the terms of the licensing agreement, the user will see a data input screen (Figure 7-1), which includes the following fields and menu choices:

1. **Project Information:** Enter sufficient information to describe the project or portions of the project for which the current data sheet applies, keeping in mind that there may be several data sheets for the same project.

2. **Pipe Type:** Select ECP, LCP or BWP. If the type of PCCP is not known, choose LCP up to and including 48-inch diameter, or ECP for 54-inch diameters and greater.

3. **Pipe Properties:** Select the pipe diameter from the drop-down menu. Typical dimensions for the steel cylinder outside diameter (OD), core thickness and joint diameter will be shown. Alternate dimensions may be entered for these properties, if known. Enter the mortar coating thickness (t_m) and steel cylinder thickness (t_y). If these thicknesses are not known, enter the minimum thickness for each, 1.0 inch and 0.0598 inch, respectively.

   0.0598" MIN. PER AWWA C301

4. **Pressure:** Enter the working pressure and field test pressure.

   PER CMS SPEC

5. **Soil Information:** Choose the most representative soil type from the five (5) options provided in the drop-down menu. The soil types (I, II, III, IV and V) are described in *AWWA Manual M9*, Table 9-1, "Soil Type Selection Guide." If different soil types are encountered along the pipe alignment, use a different input sheet for each type. As noted in Chapter 5, calculations for submerged pipe and soil cannot be derived using TRDP at this time.

   TYPE 5 IS MOST CONSERVATIVE

6. **Material Properties:** Enter the minimum yield strength of the steel cylinder (f_y) and the minimum 28-day compressive strength of the core (f'_c).

Unless otherwise specified, the following values should be used:

TYPE OF PIPE	f_y	f'_c
ECP	36,000 psi	4500 psi
LCP	36,000 psi	4,500 psi (cast) 6,000 psi (spun)
BWP	36,000 psi	4,500 psi

7. **Joint Type:** Select the type of restrained joint: welded or mechanically harnessed.

8. **Units:** Currently only US Customary Units are available.

9. **Bend Information:** Up to six (6) bends may be designed per sheet. Use additional sheets as necessary. Enter a station or other identification for each bend to be analyzed. Choose the bend angle from the drop down menu or enter a specific angle, if known. For bulkhead conditions (test bulkheads, tees, etc.), enter an angle of zero (0). Enter the length of the bend, if known, or contact a local pipe manufacturer for assistance.

   Enter the minimum length of the first pipe adjacent to the bend and the length of typical standard pipe. If this information is not known, use 20 feet for ECP and LCP and 24 feet for BWP, for both the first and typical lengths. Enter the depth of soil cover. Bends of the same angle with differing depths of soil cover may be designed on subsequent lines.

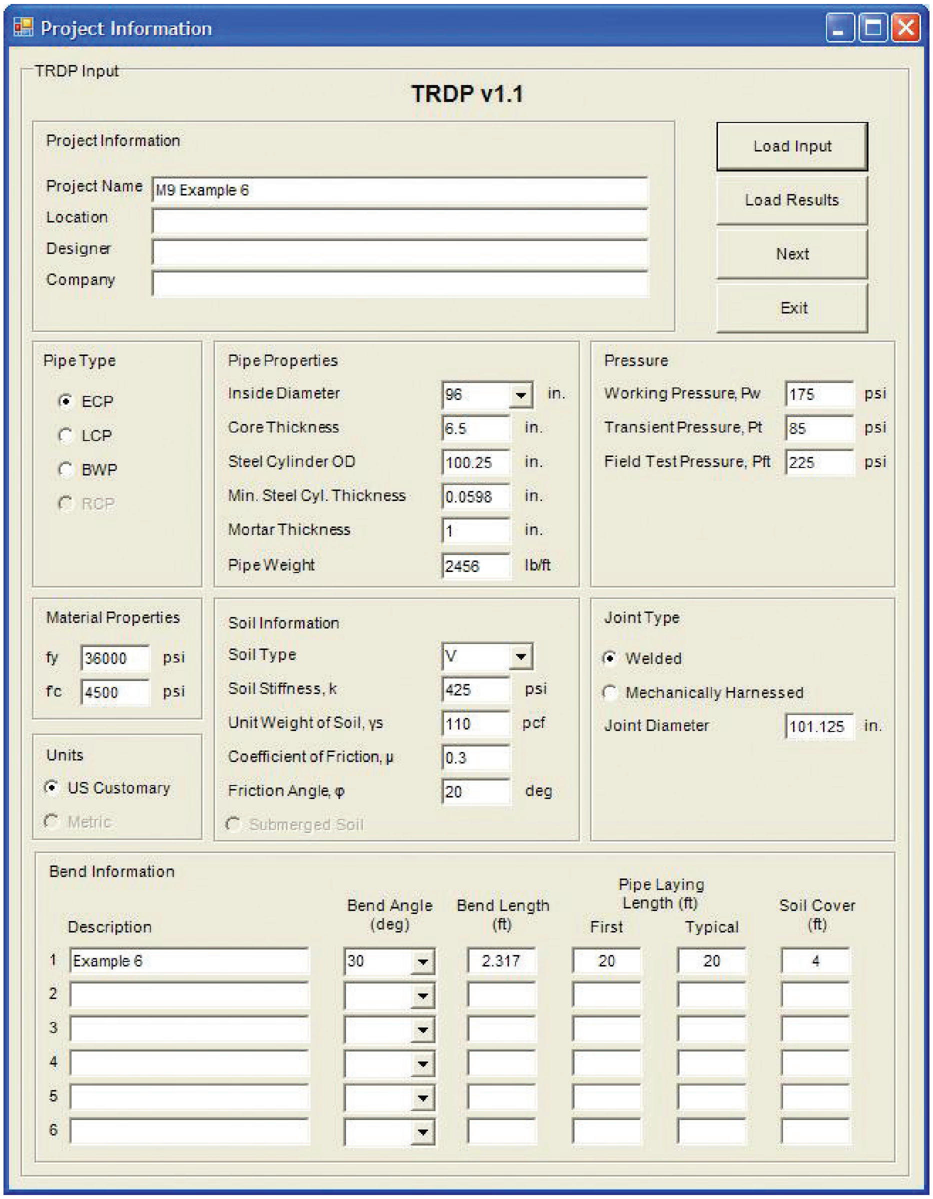
10. **Select "Next"** to begin the design process.

BEND LENGH 3.0 FT. FOR 11.25, 22.4, AND 45 BENDS PER AMERICAN DUCTILE PIPE MANUAL, 20TH EDITION PDF PG 135

INPUT & OUTPUT FILES

Project information and calculations can be saved using input and output data files. These small text files allow a user to save and access project information at any time using the "Load Input" and "Load Results" buttons on the first page of the program, as shown in Figure 7-1.

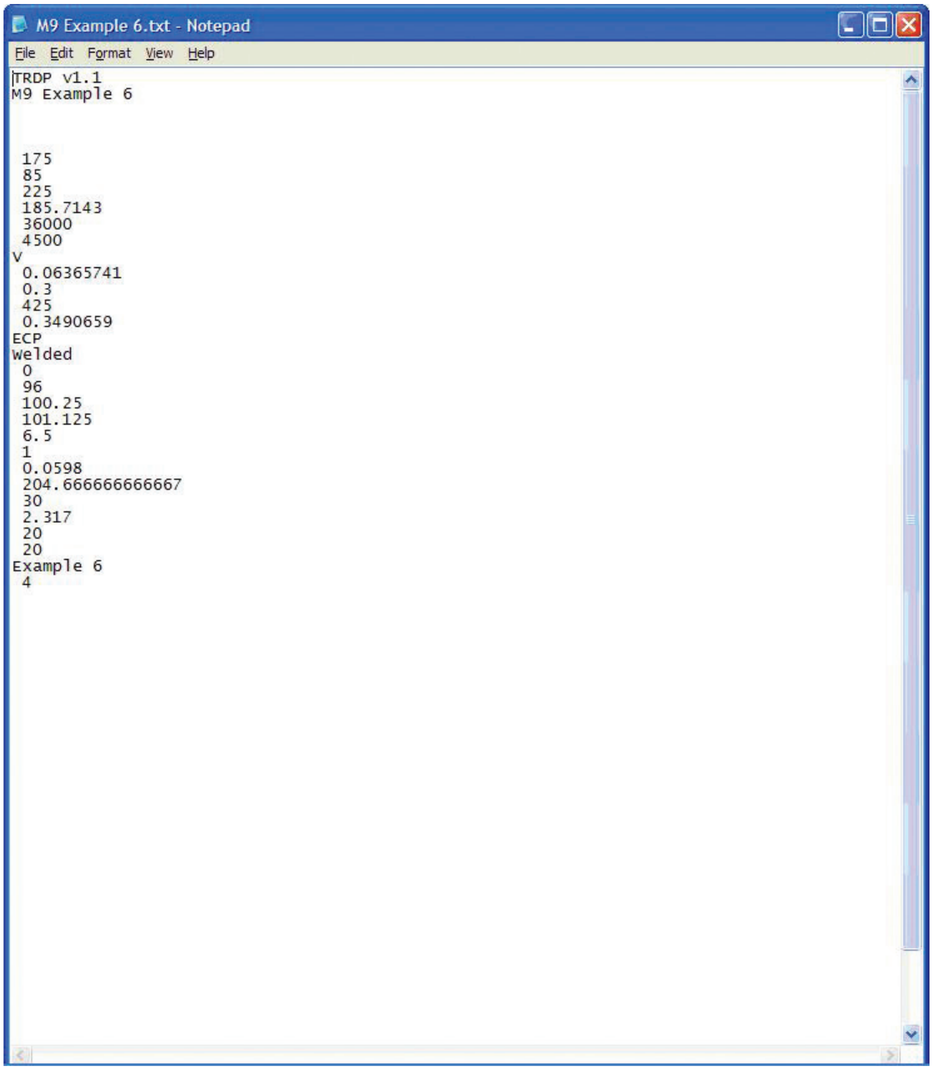
Figure 7-1



INPUT & OUTPUT FILES *(CONTINUED)*

The ***Load Input*** option allows the user to import a data input file. This file stores all project information, including project name, location, pipe type, pipe size, pressures and bend information, as previously input by the user. An example data input file is shown in Figure 7-2.

Figure 7-2



M9 Example 6.txt - Notepad

File Edit Format View Help

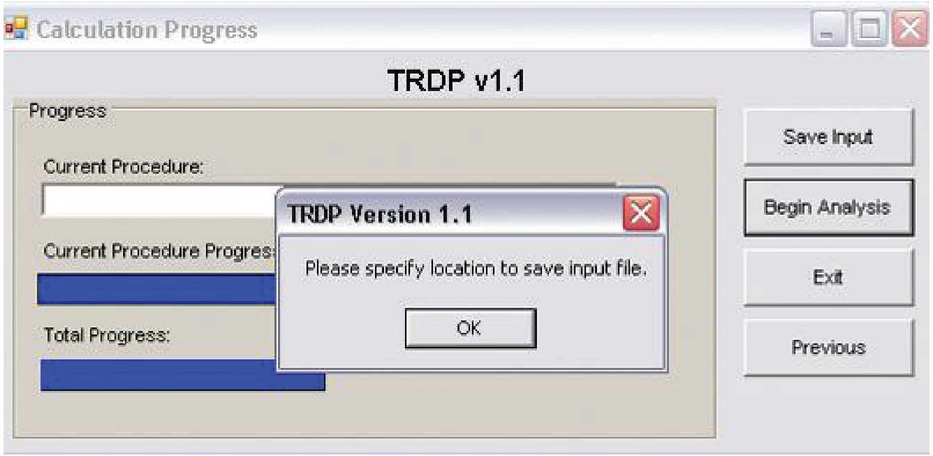
```
TRDP v1.1
M9 Example 6


 175
 85
 225
 185.7143
 36000
 4500
V
 0.06365741
 0.3
 425
 0.3490659
ECP
welded
 0
 96
 100.25
 101.125
 6.5
 1
 0.0598
 204.666666666667
 30
 2.317
 20
 20
Example 6
 4
```

INPUT & OUTPUT FILES *(CONTINUED)*

To load an input file, simply click on the ***Load Input*** button on the project information screen. This will open a dialog box allowing the user to search for and open a previously saved input file. Using this process, project information can be changed as necessary.

Once all changes to the project information have been completed, the user will be asked to save a copy of the new input file before beginning a new analysis (as shown in Figure 7-3).

Figure 7-3

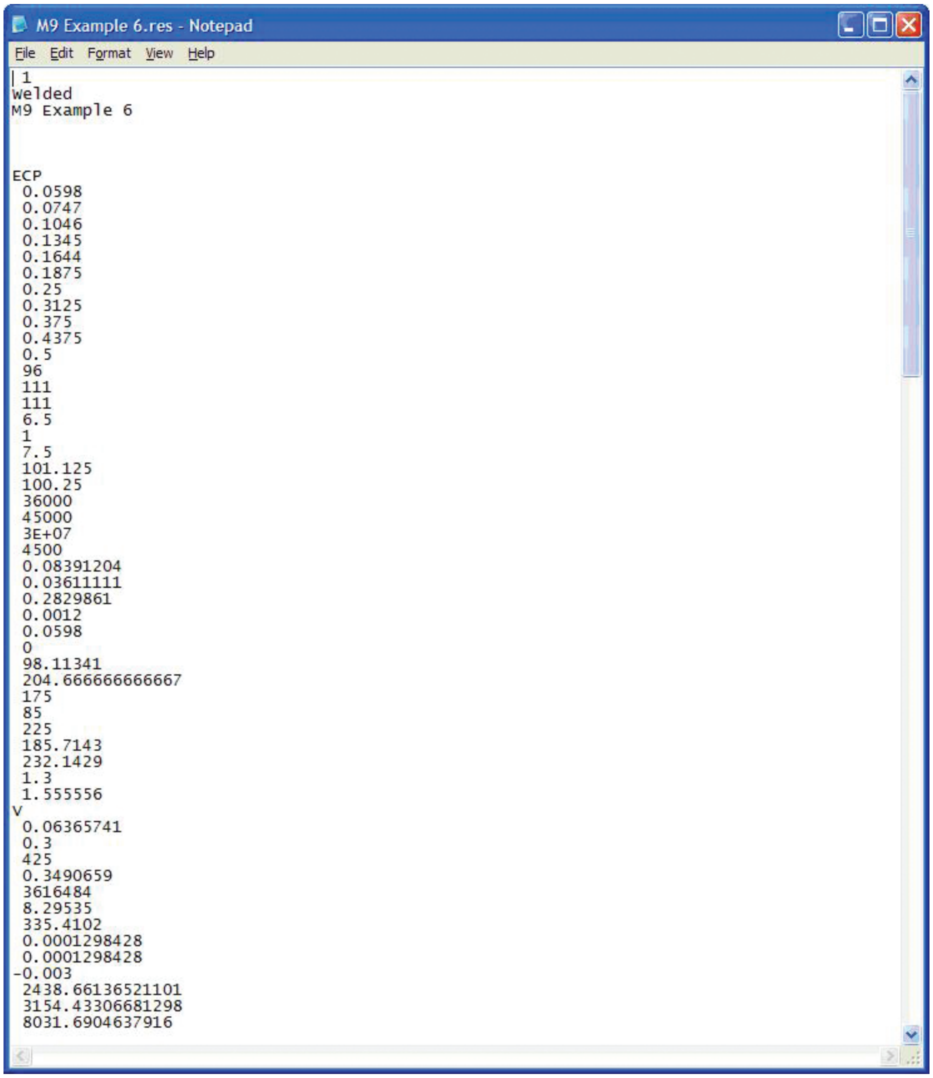


INPUT & OUTPUT FILES *(CONTINUED)*

The ***Load Results*** option allows the user to import a data output file. To do this, click on the ***Load Results*** button on the project information screen. This will open a dialog box allowing the user to search for a previously saved output file. The output file stores all results for each bend calculation for a project. The output file is in a RES format, which can be viewed using Notepad as shown in Figure 7-4.

Upon selection of an output data file, users will be directly taken to the final results screen in TRDP, where they can print or view any previously completed thrust calculations.

Figure 7-4




```
1
welded
M9 Example 6


ECP
 0.0598
 0.0747
 0.1046
 0.1345
 0.1644
 0.1875
 0.25
 0.3125
 0.375
 0.4375
 0.5
 96
 111
 111
 6.5
 1
 7.5
 101.125
 100.25
 36000
 45000
 3E+07
 4500
 0.08391204
 0.03611111
 0.2829861
 0.0012
 0.0598
 0
 98.11341
 204.666666666667
 175
 85
 225
 185.7143
 232.1429
 1.3
 1.555556
V
 0.06365741
 0.3
 425
 0.3490659
 3616484
 8.29535
 335.4102
 0.0001298428
 0.0001298428
-0.003
 2438.66136521101
 3154.43306681298
 8031.6904637916
```

INTERPRETING DATA

A summary sheet of the input and output data is shown in Figure 8-1. User inputs are highlighted in yellow. Results of the calculations are shown in the tables at the bottom of the summary sheet.

The first table shows the calculated thrust force in kip (kilopound force), total restrained length and total length of heavy-gauge cylinder required on one side of the elbow.

Figure 8-1

PROGRAM TRDP v1.1
PROJECT M9 Example 6
LOCATION
DESIGNED BY
CHECKED BY
COMPANY
DESCRIPTION Example 6

Sheet no. 1 of 1
Date 11/4/2011

Pipeline Information

Pipe Type (ECP, LCP, BWP, RCP)	ECP	
Internal Diameter, ID	96	in
Core Thickness, XX	6.5	in
Mortar Coating Thickness, XX	1	in
Core Outside Diameter, OD	109	in
Pipe Outside Diameter, XX	111	in

Joint Properties

Joint Type (Welded or Harnessed)	Welded	
Joint Diameter, XX	101 125	in
Joint Slack	0	in

Pressures

Working Pressure, XX	175	psi
Transient Pressure, XX	85	psi
Field Test Pressure, XX	225	psi
XXX	186	psi

Cylinder Outside Diameter, XX	100.25	in
Minimum Cylinder Thickness	0.0598	in
Bend Angle, Δ	30	deg
Centerline Length of Fitting, XX	2.32	ft
Pipe Laying Length (First Pipe), XX	20	ft
Pipe Laying Length (Typical Pipe), XX	20	ft

Material Properties

Concrete Strength, XX	4500	psi
Steel Cylinder Yield Strength, XX	36000	psi

Soil Information
(Table 9-1 - Soil Type Selection Guide)

Soil Type (I through V)	V	
Soil Stiffness, k	425	psi
Soil Unit Weight, XX	110	pcf
Pipe to Soil Friction Coefficient, XX	0.3	in
Soil Cover, H	4	ft
Angle of Internal Friction, XX	20	deg

Bend Angle (deg)	Centerline Length of Fitting (ft)	Thrust (kip)	Total Footage Required (one side)	Total Heavy Gage Footage (one side)
30	2.32	985	274	213

Required Lengths for One Side

Cylinder Thickness (in.)	0.5	0.4375	0.375	0.3125	.25	0.1875	0.1644 (8 GA)	0.1345 (10 GA)	0.1048 (12 GA)	0.0747 (14 GA)	0.0598 (16 GA)
Length Needed (ft)	0	0	0	17.8	64	23.7	30.6	30.6	30.6	15.5	61.3
Number of Pipes	0	0	0	1	4	1	1	2	1	1	3

INTERPRETING DATA *(CONTINUED)*

The second table on the summary sheet shows minimum length required for each steel cylinder thickness. Normally, a project's pipe-laying schedule should show equal or longer pipe length for each cylinder thickness in the table, although an additional length of a thicker cylinder pipe can be substituted for the thinner cylinder pipe length. This may result in shorter pipe length in some cylinder thicknesses. Using the example above, the actual layout of the 96-inch pipe on one side of the 30° elbow may have the following pipe lengths:

One (1) 20-foot length with 5/16 inch (0.3125 inch) cylinder

Four (4) 20-foot lengths with 1/4 inch (0.2500 inch) cylinder

One (1) 20-foot length with 3/16 inch (0.1875 inch) cylinder

One (1) 20-foot length with 8 GA (0.1644 inch) cylinder

Two (2) 20-foot lengths with 10 GA (0.1345 inch) cylinder

One (1) 20-foot length with 12 GA (0.1046 inch) cylinder

One (1) 20-foot length with 14 GA (0.0747 inch) cylinder

Three (3) 20-foot lengths with 16 GA (0.0598 inch) cylinder

Table 8-1 compares the pipe lengths used in actual layout with the calculated pipe lengths for each cylinder thickness. You may note that the highlighted pipe lengths on the third row of the table are shorter compared to calculated pipe lengths shown on the second row for the same cylinder thickness. This difference in lengths results from the extra pipe length required for thicker cylinder pipe, the figures for which appear in the fourth row of the column to the left of each actual pipe length. In general, the sum of the lengths shown on the third row and on the fourth row in the previous column should be equal or greater than the pipe length shown on the second row for a particular cylinder thickness.

Other pipe length combinations may be applied based on the pipe manufacturer's preference. The user should carefully examine available pipe lengths for each cylinder thickness and compare it with calculated lengths needed for each cylinder thickness listed on the results table.

Table 8-1

Sum of these two lengths > required 23.7'

Cylinder Thickness (in.)	0.3125	.25	0.1875	0.1644 (8 GA)	0.1345 (10 GA)	0.1048 (12 GA)	0.0747 (14 GA)	0.0598 (16 GA)	Total Pipe Length (ft)
Length Needed (ft)	17.8	64	23.7	30.6	30.6	30.6	15.3	61.3	273.9
Actual Pipe Length (ft)	20	80	20	20	40	20	20	60	280
Extra Length Applied to Thinner Cyl. (ft)	2.2	18.2	14.5	3.9	13.3	2.7	7.4	6.1	

APPENDIX

DESIGN EXAMPLES

Example design calculations are presented for three different pipe types, namely, two bend angles and a bulkhead. The design conditions, pipe properties and installation parameters used for each of these examples are presented in **Table A-1**. The input and results from five additional examples are provided in **Table A-2** without accompanying calculations. These results were obtained from the Thrust Restraint Design Program.

The pipe material properties and characteristics presented in **Tables A-1** and **A-2** have been selected for illustrative purposes and should not be used as actual design values without verification of their appropriateness.

DESIGN EXAMPLE 1
36 in. Diameter LCP (ANSI/AWWA 301) — Bulkhead

1. Determine effective working pressure:

$$P_{weff} = \max\left(\frac{P_{ft}}{1.25}, \frac{P_w + P_t}{1.4}\right) = \max\left(\frac{200}{1.25}, \frac{150+80}{1.4}\right)$$

$$= \max(160, 164) = 164\text{psi} < 200\text{psi} = P_{ft}$$

Therefore, use P_{ft} to calculate cylinder thickness required.

2. Determine thrust in pipe:

Pressurized area of pipe:

$$A = \pi\frac{D_j^2}{4} = \pi\frac{(41\text{ in.})^2}{4} = 1,320\text{in.}^2$$

Thrust in pipe at P_{ft}:

$$F = P_{ft}A = 200\text{ psi} \times 1,320\text{in.}^2 = 264\text{ kip}$$

3. Determine the pipe cylinder area requirement at the bulkhead:

Required pipe cylinder area:

$$A_{yo} = \frac{F}{0.5f_y} = \frac{264\text{ kip}}{0.5(36\text{ ksi})} = 14.67\text{ in.}^2$$

4. Determine frictional resistance of pipe:

Weight of fluid:

$$W_{ff} = \gamma\ \frac{\pi}{4}ID^2 = (62.4\text{ pcf})\left(\frac{\pi}{4}\right)(36\text{ in.})^2 \div \left(12\frac{\text{in.}}{\text{ft}}\right)^2 = 441\frac{\text{lb}}{\text{ft}}$$

Weight of soil above pipe (Eq. 9-5B from AWWA Manual M9):

$$W_e = \gamma\left[\frac{D_oH}{12} + \left(1 - \frac{\pi}{4}\right)\left(\frac{D_o^2}{12^2 \times 2}\right)\right]$$

$$= 114\text{ pcf}\left[\frac{42.5\text{ in.}}{12\text{ in./ft}}(4\text{ ft}) + \left(1 - \frac{\pi}{4}\right)\frac{(42.5\text{ in.})^2}{(2)(12\text{ in./ft})^2}\right] = 1,768\frac{\text{lb}}{\text{ft}}$$

Shallow cover factor to account for reduced friction if the soil-to-soil friction is less than that of pipe-to-soil (Eq. 9-8 from AWWA Manual M9):

$$\beta = \frac{K_o \tan\phi}{\mu} \frac{\left(\frac{12H}{D_o} + 0.5\right)^2}{\left(\frac{12H}{D_o} + 0.107\right)}$$

$$= \frac{\left(1 - \sin 30^\circ\right)\tan 30^\circ}{0.5} \frac{\left[\left(\frac{4\text{ ft}}{42.5\text{ in.}}\right)\left(12\frac{\text{in.}}{\text{ft}}\right) + 0.5\right]^2}{\left[\left(\frac{4\text{ ft}}{42.5\text{ in.}}\right)\left(12\frac{\text{in.}}{\text{ft}}\right) + 0.107\right]} = 1.24 > 1 \therefore \text{use } 1$$

Soil frictional force (Eq. 9-7 from AWWA Manual M9):

$$f_\mu = \mu\left[\left(1+\beta\right)W_e + W_p + W_f\right] = 0.5\left[\left(1+1\right)\left(1{,}768\frac{\text{lb}}{\text{ft}}\right) + 404\frac{\text{lb}}{\text{ft}} + 441\frac{\text{lb}}{\text{ft}}\right] = 2{,}196\frac{\text{lb}}{\text{ft}}$$

5. Thrust dissipation length required (Eq. 9-11B from AWWA Manual M9):

$$L_{ft} = 1.1\left[\frac{1.25 P_{weff} A}{f_\mu}\right] = 1.1\left[\frac{1.25\left(164\text{ psi}\right)\left(1{,}320\text{ in.}^2\right)}{2{,}196\frac{\text{lb}}{\text{ft}}}\right] = 136\text{ ft}$$

6. Determine required cylinder thicknesses over the thrust dissipation length:

Pipe cylinder areas:

$$A_{s16ga} = \left(D_y - t_y\right)\left(\pi\right)\left(t_y\right) = \left(40.5\text{ in. œ}0.0598\text{ in.}\right)\left(\pi\right)\left(0.0598\text{ in.}\right) = 7.60\text{ in.}^2$$

$$A_{s14ga} = \left(D_y - t_y\right)\left(\pi\right)\left(t_y\right) = \left(40.5\text{ in.} - 0.0747\text{ in.}\right)\left(\pi\right)\left(0.0747\text{ in.}\right) = 9.49\text{ in.}^2$$

$$A_{s12ga} = \left(D_y - t_y\right)\left(\pi\right)\left(t_y\right) = \left(40.5\text{ in.} - 0.1046\text{ in.}\right)\left(\pi\right)\left(0.1046\text{ in.}\right) = 13.27\text{ in.}^2$$

$$A_{s10ga} = \left(D_y - t_y\right)\left(\pi\right)\left(t_y\right) = \left(40.5\text{ in.} - 0.1345\text{ in.}\right)\left(\pi\right)\left(0.1345\text{ in.}\right) = 17.06\text{ in.}^2$$

Because the required area is 14.67 in.^{2} [t_{yreq} = (14.67/17.06) × 0.1345 = 0.1157 in.], a 10 ga cylinder thickness is selected for the pipe at the bulkhead.

The pipe axial force, and therefore the required cylinder thickness, diminishes on a straight-line basis to zero at the thrust dissipation length from the bulkhead.

$$L_{10\,ga} = L_{ft} - \frac{A_{s\,12ga}}{A_{yo}} L_{ft} = 136\text{ ft } \text{œ} \frac{13.27\text{ in.}^2}{14.67\text{ in.}^2}\left(136\text{ ft}\right) = 13\text{ ft}$$

$$L_{12\,ga} = L_{ft} - L_{10\,ga} - \frac{A_{s\,14ga}}{A_{yo}} L_{ft} = 136\text{ ft } \text{œ}13\text{ ft } \text{œ} \frac{9.49\text{ in.}^2}{14.67\text{ in.}^2}\left(136\text{ ft}\right) = 35\text{ ft}$$

$$L_{14\,ga} = L_{ft} - L_{10\,ga} - L_{12\,ga} - \frac{A_{s\,16ga}}{A_{yo}} L_{ft} = 136\text{ ft } \text{œ}13\text{ ft } \text{œ}35\text{ ft } \text{œ} \frac{7.60\text{ in.}^2}{14.67\text{ in.}^2}\left(136\text{ ft}\right) = 18\text{ ft}$$

$$L_{16\,ga} = L_{ft} - L_{10\,ga} - L_{12\,ga} - L_{14\,ga} = 136\text{ ft } \text{œ}13\text{ ft } \text{œ}35\text{ ft } \text{œ}18\text{ ft} = 70\text{ ft}$$

See Figure A-11 for the minimum restrained footage and minimum cylinder thickness requirements for this design example. Typical restrained footages and cylinder thicknesses based on the use of 20 ft lengths are also shown.

The minimum restrained joint strength (F_j) required without material failure or joint leakage is calculated from Eq. 9-15 from AWWA Manual M9:

$$F_j = \pi\left(D_y - t_{yreq}\right)t_{yreq} f_y = A_{yo} f_y$$

Where:

$$F_j = \left(14.67\text{ in.}^2\right)\left(36{,}000\text{ psi}\right)$$

DESIGN EXAMPLE 2

54 in. Diameter BWP (ANSI/AWWA C303) – 45° Bend, Welded Joints

1. The combination of bending and axial tension in a restrained pipe adjacent to a bend requires an iterative solution for the cylinder thickness. Select a trial cylinder thickness equal to the minimum thickness needed for proper cylinder/reinforcing bar balance in the design for internal pressure. For this example, choose t_y = 0.1875 in.

2. Ultimate strength of the pipe in bending and axial tension is calculated conservatively assuming a bilinear stress-strain relationship for steel, a Whitney block stress-strain diagram for concrete in compression and neglecting the tensile strength of concrete or mortar.

   Ultimate tensile strength of the pipe:

$$F_{ultimate} = \left(D_y - t_y\right)\left(\pi\right)\left(t_y\right)\left(f_y\right) = \left(55.875\text{ in. } \text{œ}\ 0.1875\text{ in.}\right)\left(\pi\right)\left(0.1875\text{ in.}\right)\left(36\text{ ksi}\right) = 1181\text{ kip}$$

Ultimate moment strength of the pipe is determined by calculating the neutral axis location to achieve a net axial force of zero while setting the maximum strain in the concrete to the compressive limit of ε_u = 0.003 (see Figure 9-12 from AWWA Manual M9). The bilinear stress-strain relationship for steel is used, limiting the tensile strength of steel to f_y. Calculating force equilibrium results in a neutral axis, y = 15.145 in. from the pipe edge on the compression side. Using the concrete stress block of depth = β_{1y}, where β_1 = 0.825 for 4,500 psi concrete (as defined in ACI 318), and a concrete stress of 0.85 f'_c , the resulting ultimate moment strength, $M_{ultimate}$ = 27,921 kip-in.

3. Yield strength of the pipe in bending and axial tension is calculated from the maximum strength of the reinforced concrete pipe cross section with bilinear steel and trilinear concrete stress-strain relationships. The maximum axial force strength may occur at the onset of concrete microcracking and prior to the onset of steel cylinder yielding.

Area of mortar outside the steel cylinder:

$$A_{co} = 0.25\pi\left[D_o^{\,2} - D_y^{\,2}\right] = 0.25(\pi)\left(58.375^2 - 55.875^2\right) = 224.3\,\text{in.}^2$$

Tensile strain limits for concrete stress-strain relationships are:

$$\varepsilon_t = \frac{7\sqrt{f'_c}}{E_c} = \frac{7\sqrt{4,500}}{3,616,500} = 0.0001298$$

$$\varepsilon_k = 11\varepsilon_t = 11(0.0001298) = 0.001428$$

Because $\varepsilon_k > \varepsilon_y$, concrete strength at steel yield strain:

$$f_{cy} = \frac{7\sqrt{f'_c}}{\varepsilon_t - \varepsilon_k}\left(\varepsilon_y - \varepsilon_k\right) = \frac{7\sqrt{4,500}}{0.0001298 - 0.001428}\left(0.0012 - 0.001428\right) = 0.0825\text{ ksi}$$

Tensile strength of pipe at onset of steel cylinder yielding:

$$F_{yield} = A_{co} f_{cy} + (D_y - t_y)(\pi)(t_y)(f_y)$$
$$= \left(224.3\text{ in.}^2\right)\left(0.0825\text{ ksi}\right) + (55.875\text{ in. - }0.1875\text{ in.})(\pi)\left(0.1875\text{ in.}\right)\left(36\text{ ksi}\right) = 1199\text{ kip}$$

Yield moment strength of the pipe is determined by calculating the neutral axis location to achieve a net axial force of zero while setting the maximum strain in the outermost steel to the yield limit of ε_y = 0.0012 (see Figure 9-11 from AWWA Manual M9). The bilinear stress-strain relationship for steel and the trilinear concrete tensile properties were used. Calculating force equilibrium results in a neutral axis, y = 22.402 in. from the pipe edge on the compression side. The resulting yield moment strength, M_{yield} = 19,611 in.-kip.

4. Determine pressure induced thrust and pipe properties.

Pressurized area of pipe:

$$A = \pi \frac{D_j^2}{4} = \pi \frac{(56.375 \text{ in.})^2}{4} = 2{,}496 \text{ in.}^2$$

Thrust in pipe at P_{fteff} = 1.25 $_{Pweff}$:

$$T = 2(1.25)(P_{weff})(A)\left(\sin\frac{\Delta}{2}\right) = 2(1.25)(214 \text{ psi})(2{,}496 \text{ in.}^2)\left(\sin\frac{45°}{2}\right) = 511.7 \text{ kip}$$

Weight of fluid:

$$W_{ff} = \gamma\ \frac{\pi}{4} ID^2 = 62.4 \text{ pcf}\left(\frac{\pi}{4}\right)(54 \text{ in.})^2 \div \left(12\frac{\text{in.}}{\text{ft}}\right)^2 = 992\frac{\text{lb}}{\text{ft}}$$

Weight of soil above pipe (Eq. 9-5B from AWWA Manual M9):

$$W_e = \gamma\left[\frac{D_oH}{12} + \left(1-\frac{\pi}{4}\right)\left(\frac{D_o^2}{12^2 \times 2}\right)\right]$$

$$= 120 \text{ pcf}\left[\frac{58.375 \text{ in.}}{12 \text{ in./ft}}(6 \text{ ft}) + \left(1-\frac{\pi}{4}\right)\frac{(58.375 \text{ in.})^2}{2(12 \text{ in./ft})^2}\right] = 3{,}807\frac{\text{lb}}{\text{ft}}$$

Shallow cover factor is expressed by Eq. 9-8 from AWWA Manual M9 as:

$$\beta = \frac{K_o \tan\phi}{\mu}\frac{\left(\frac{12H}{D_o} + 0.5\right)^2}{\left(\frac{12H}{D_o} + 0.107\right)} \le 1$$

$$= \frac{\left[1-\sin(34°)\right]\tan(34°)}{0.5}\frac{\left[\frac{6 \text{ ft}}{58.375 \text{ in.}}\left(12\frac{\text{in.}}{\text{ft}}\right) + 0.5\right]^2}{\frac{6 \text{ ft}}{58.375 \text{ in.}}\left(12\frac{\text{in.}}{\text{ft}}\right) + 0.107} = 1.33 > 1 \therefore \text{ use } 1.0$$

Soil frictional force (Eq. 9-7 from AWWA Manual M9):

$$f_\mu = \mu\left[(1+\beta)W_e + W_p + W_f\right] = 0.5\left[(1+1)\left(3{,}807\frac{\text{lb}}{\text{ft}}\right) + 518\frac{\text{lb}}{\text{ft}} + 992\frac{\text{lb}}{\text{ft}}\right] = 4{,}562\frac{\text{lb}}{\text{ft}}$$

Steel cylinder area:

$$A_y = \pi(D_y - t_y)(t_y) = \pi(55.875 \text{ in.} - 0.1875 \text{ in.})(0.1875 \text{ in.}) = 32.8 \text{ in.}^2$$

Concrete wall area:

$$A_c = 0.25\pi(D_o^2 - ID^2) - A_y = 0.25\pi\left[(58.375 \text{ in.})^2 - (54 \text{ in.})^2\right] - 32.8 \text{ in.}^2 = 353 \text{ in.}^2$$

Transformed pipe wall area:

$$A_t = A_c + nA_y = 353\text{ in.}^2 + 8.295\left(32.8\text{ in.}^2\right) = 625\text{ in.}^2$$

Steel cylinder moment of inertia:

$$I_s = \frac{\pi}{64}\left[D_y^{\ 4} - \left(D_y - 2t_y\right)^4\right] = \frac{\pi}{64}\left\{\left(55.875\text{ in.}\right)^4 - \left[55.875\text{ in.} - 2\left(0.1875\text{ in.}\right)\right]^4\right\} = 12{,}716\text{ in.}^4$$

Effective transformed (to concrete) pipe moment of inertia, including softening factor:

$$I_{eff} = \left[\frac{\pi}{64}\left(D_o^{\ 4} - ID^4\right) - I_s\right]\psi + nI_s$$
$$= \left[\frac{\pi}{64}\left[\left(58.375\text{ in.}\right)^4 - \left(54\text{ in.}\right)^4\right] - 12{,}716\text{ in.}^4\right]\left(0.2\right) + \left(8.295\right)\left(12{,}716\text{ in.}^4\right) = 133{,}458\text{ in.}^4$$

Characteristic beam on elastic foundation length (Eq. 9-10 from AWWA Manual M9):

$$\lambda = \sqrt[4]{\frac{k}{4E_cI_{eff}}} = \sqrt[4]{\frac{3{,}400\text{ lb/in./in.}}{4\left(3{,}616{,}500\text{ psi}\right)\left(133{,}458\text{ in.}^4\right)}} = 0.006478\frac{1}{\text{in.}}$$

5. Substituting Eq. 9-9A, 9-9B, and 9-11A into Eq. 9-6 from AWWA Manual M9, the quadratic equation is solved:

$$aF_o^{\ 2} + bF_o + c = 0$$

F_o = axial force at the first joint is given by:

$$F_o = \frac{-b + \sqrt{b^2 - 4ac}}{2a}\text{lbf}$$

$$= \frac{-2.517\times10^{11} + \sqrt{\left(2.517\times10^{11}\right)^2 - 4\left(5.253\times10^5\right)\left(-1.683\times10^{17}\right)}}{2\left(5.253\times10^5\right)}\text{lbf} = 375.1\text{ kip}$$

Where:

$$a = \left[\frac{k}{\lambda}\left(\cos\frac{\Delta}{2}\right)^2 + kl_b\left(\cos\frac{\Delta}{2}\right)\right]\frac{\text{in.}}{\text{lb}}$$

$$= \left[\frac{3{,}400 \text{ lb/in./in.}}{0.006478/\text{in.}}\left(\cos\frac{45°}{2}\right)^2 + (3{,}400 \text{ lb/in./in.})(24.6 \text{ in.})\left(\cos\frac{45°}{2}\right)\right]\frac{\text{in.}}{\text{lb}} = 5.253\times10^5$$

$$b = \left[2f_\mu E_c A_t\left(\sin\frac{\Delta}{2}\right)^2\right]\frac{\text{in.}}{\text{lb}^2}$$

$$= \left[2\left(4{,}562\frac{\text{lb}}{\text{ft}}\right)\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)(3{,}616{,}500 \text{ psi})(625 \text{ in.}^2)\left(\sin\frac{45°}{2}\right)^2\right]\frac{\text{in.}}{\text{lb}^2} = 2.517\times10^{11}$$

$$c = \left[-E_c A_t f_\mu\left(\sin\frac{\Delta}{2}\right)T\right]\frac{\text{in.}}{\text{lb}^3}$$

$$= \left[-(3{,}616{,}500 \text{ psi})(625 \text{ in.}^2)\left(4{,}562\frac{\text{lb}}{\text{ft}}\right)\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)\left(\sin\frac{45°}{2}\right)(511{,}700 \text{ lb})\right]\frac{\text{in.}}{\text{lb}^3}$$

$$= -1.683\times10^{17}$$

The length of pipe required to resist or dissipate the axial force:

$$L_{ft} = \frac{F_o}{f_\mu} = \frac{375.1 \text{ kip}}{4.56\frac{\text{kip}}{\text{ft}}} = 82.3 \text{ ft}$$

Moment at the first joint:

$$M_o = \frac{F_o k L_{ft}}{4\lambda^2 A_t E_c \tan\frac{\Delta}{2}}$$

$$= \frac{(375{,}100 \text{ lb})(3{,}400 \text{ lb/in./in.})\left[(82.3\times12) \text{ in.}\right]}{4\left(\frac{0.006478}{\text{in.}}\right)^2(625 \text{ in.}^2)(3{,}616{,}500 \text{ psi})\left(\tan\frac{45°}{2}\right)} = 8{,}014 \text{ in. -kip}$$

The axial force, F_o, and the moment, M_o, calculated previously are at a distance, l_b, from the P.I. of the bend. The axial force, F, and moment, M, used to calculate the required cylinder thickness must be evaluated where the restrained joint is attached to the steel cylinder of the first pipe adjacent to the bend. As with l_b, the extra distance from the P.I. to the attachment point will vary among manufacturers — for this design example, the extra distance is assumed to be 6 in. At the attachment point, F = 373 kips and M = 7,392 in.-kip. These results are then compared to the moment interaction diagram to determine if the design is acceptable. The interaction diagrams for the ultimate strength of the pipe cross section and the onset of yielding of the steel cylinder are both nonlinear, but have been approximated here by straight lines connecting the tensile strength at zero moment to the moment capacity at zero tension for each criterion in order to simplify the calculations for the design examples. The reduced interaction lines connect the maximum tensile strength at zero moment divided by the load factor with the maximum moment strength at zero tension divided by the load factor for each criterion. As can be seen on the interaction diagram, Figure A-1, this design, represented by point D, lies below both factored interaction lines and thus meets the design requirements.

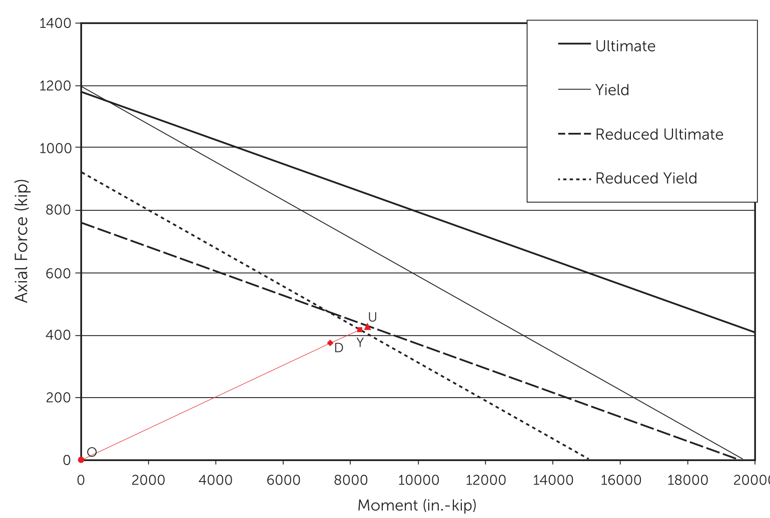
The margin of safety against the ultimate strength or the yield strength is a measure of the amount by which the pipe strength exceeds the required pipe strength, including all factors of safety, at the connection of the restrained joint to the first pipe cylinder. The excess strength may be determined by dividing the length of line OU, for ultimate strength, or line OY, for yield strength, by the length of line OD. The margin of safety, $MS_{ultimate}$, is calculated from the formula:

$$MS_{ultimate} = \frac{1}{\frac{M}{M_{ultimate}} + \frac{F}{F_{ultimate}}}\left(\frac{1}{LF}\right)$$

The margin of safety against the ultimate strength criterion is:

$$MS_{ultimate} = \frac{1}{\frac{M}{M_{ultimate}} + \frac{F}{F_{ultimate}}}\left(\frac{1}{1.56}\right) = \frac{1}{\frac{7,392 \text{ in. kip}}{27,921 \text{ in. kip}} + \frac{373 \text{ kip}}{1,181 \text{ kip}}}\left(\frac{1}{1.56}\right) = 1.10$$

Figure A-1 – Interaction Diagram for Example 2



The axial force-moment interaction diagrams corresponding to the ultimate strength and the onset of yielding of the steel cylinder are both nonlinear, but have been approximated by a linear relationship here for ease of computation of the examples.

The margin of safety against the yield criterion is:

$$MS_{yield} = \frac{1}{\frac{M}{M_{yield}} + \frac{F}{F_{yield}}}\left(\frac{1}{1.3}\right) = \frac{1}{\frac{7,392 \text{ in. kip}}{19,671 \text{ in. kip}} + \frac{373 \text{ kip}}{1,199 \text{ kip}}}\left(\frac{1}{1.3}\right) = 1.12$$

The minimum cylinder thickness required is therefore

$$t_{yreq} = \frac{0.1875 \text{ in.}}{1.12} = 0.1674 \text{ in.}$$

The 0.1875 in. value must be used because of the minimum cylinder thickness requirements used for this design example (Table A-1).

The required restrained joint strength (F_j) to resist material failure or joint leakage is calculated from Eq. 9-15 from AWWA Manual M9:

$$\begin{aligned} F_j &= \pi\left(D_y - t_{yreq}\right)t_{yreq}f_y \\ &= \pi\left(55.875 \text{ in.} - 0.1674 \text{ in.}\right)\left(0.1674 \text{ in.}\right)\left(36,000 \text{ psi}\right) \\ &= 1,054,700 \text{ lb} \end{aligned}$$

DESIGN EXAMPLE 3

42 in. Diameter ECP (ANSI/AWWA C301) – 75° Bend, Harnessed Joints

The following work does not show all calculations. For example, determination of the neutral axis, pipe capacities, and the solution method for solving ten equations simultaneously are standard procedures, and the details are omitted for brevity.

Example 3 uses an iterative solution whereby a solution is found for at least two axial restraint lengths selected by iteration to envelope the solution. The final solution is determined by interpolation between bounding results.

1. The combination of bending and axial tension in restrained pipe adjacent to a bend requires an iterative solution for the cylinder thickness and for the number of restrained pipe. The use of mechanically harnessed joints demands a segmented beam on elastic foundation solution as described in the closure to the paper by Zarghamee et al. (2004). Select a trial cylinder thickness, t_y = 0.1046 in. (12 ga).

2. The ultimate strength of the pipe in bending and axial tension is calculated conservatively assuming a Whitney block stress-strain diagram for concrete in compression and neglecting the tensile strength of concrete.

Pipe axial tensile strength at yield of the steel cylinder is:

$$F_{ultimate} = \left(D_y - t_y\right)\left(\pi\right)\left(t_y\right)\left(f_y\right) = \left(44.5\text{ in.} - 0.1046\text{ in.}\right)\left(\pi\right)\left(0.1046\text{ in.}\right)\left(36\text{ ksi}\right) = 525\text{ kip}$$

Moment strength of pipe is determined by calculating the neutral axis location to achieve a net axial force of zero while setting the maximum strain in the concrete to the compressive limit of ε_u = 0.003. The bilinear stress-strain relationship for steel is used, limiting tensile stress to f_y. Calculating force equilibrium results in a neutral axis, y = 6.58 in. from the pipe edge on the compression side. Using the concrete stress block of depth = β_{1y}, where β_1 = 0.825 for 4,500 psi concrete as defined in ACI 318, and a concrete stress of 0.85 f'_c, the resulting ultimate moment strength, $M_{ultimate}$ = 11,745 in.-kip.

3. Yield strength of the pipe in bending and axial tension is calculated from the maximum strength of the reinforced concrete pipe cross section with bilinear steel and trilinear concrete stress-strain relationships. The maximum axial tensile strength may occur at the onset of concrete microcracking, prior to the onset of steel cylinder yielding.

Area of concrete:

$$A_c = 0.25\pi\left(D_o^2 - ID^2\right) - t_y\left(\pi\right)\left(D_y - t_y\right)$$
$$= 0.25\pi\left[\left(51\text{ in.}\right)^2 - \left(42\text{ in.}\right)^2\right] - 0.1046\text{ in.}\left(\pi\right)\left(44.5\text{ in.} - 0.1046\text{ in.}\right) = 643\text{ in.}^2$$

Area of outer pipe wall (inner core assumed to have no tensile strength):

$$A_{co} = 0.25\pi\left[D_o^2 - D_y{}^2\right] = 0.25\left(\pi\right)\left(51\text{ in.}^2 - 44.5\text{ in.}^2\right) = 488\text{ in.}^2$$

Tensile strength of pipe at onset of steel cylinder yielding:

$$F_{yield} = A_{co}f_{cy} + \left(D_y - t_y\right)\left(\pi\right)\left(t_y\right)\left(f_y\right)$$
$$= \left(488\text{ in.}^2\right)\left(0.0824\text{ ksi}\right) + \left(44.5\text{ in.} - 0.1046\text{ in.}\right)\left(\pi\right)\left(0.1046\text{ in.}\right)\left(36\text{ ksi}\right) = 565\text{ kip}$$

Yield moment strength of the pipe is determined by calculating the neutral axis location to achieve a net axial force of zero while setting the maximum strain in the outermost steel to the onset of yielding of ε_y = 0.0012. The bilinear stress-strain relationship for steel and the trilinear concrete tensile properties were used. Calculating force equilibrium results in a neutral axis, y = 14.595 in. from the pipe edge on the compression side. The resulting yield moment strength M_{yield} = 9,221 in.-kip.

4. Determine pipe properties:

Pressurized area of pipe:

$$A = \pi\frac{D_j{}^2}{4} = \pi\frac{\left(45\text{ in.}\right)^2}{4} = 1{,}590\text{ in.}^2$$

Weight of fluid:

$$W_{ff} = \gamma\ \frac{\pi}{4} ID^2 = (62.4\ \text{pcf})\left(\frac{\pi}{4}\right)(42\ \text{in.})^2\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 600\frac{\text{lb}}{\text{ft}}$$

Weight of soil above pipe (Eq. 9-5B from AWWA Manual M9):

$$W_e = \gamma\left[\frac{D_o H}{12} + \left(1-\frac{\pi}{4}\right)\left(\frac{D_o^2}{12^2 \times 2}\right)\right]$$

$$= (114\ \text{pcf})\left[\frac{51\ \text{in.}}{12\ \text{in./ft}}(5\ \text{ft}) + \left(1-\frac{\pi}{4}\right)\frac{(51\ \text{in.})^2}{2(12\ \text{in./ft})^2}\right] = 2{,}643\frac{\text{lb}}{\text{ft}}$$

Shallow cover is expressed by Eq. 9-8 from AWWA Manual M9 as:

$$\beta = \frac{K_o \tan\phi}{\mu}\frac{\left(\frac{12H}{D_o}+0.5\right)^2}{\left(\frac{12H}{D_o}+0.107\right)} \le 1$$

$$= \frac{[1-\sin(30°)]\tan(30°)}{0.5}\frac{\left(\frac{60\ \text{in.}}{51\ \text{in.}}+0.5\right)^2}{\left(\frac{60\ \text{in.}}{51\ \text{in.}}+0.107\right)} = 1.26 > 1\ \therefore\ \text{use}\ 1.0$$

Soil frictional force (Eq. 9-7 from AWWA Manual M9):

$$f_\mu = \mu\left[(1+\beta)(W_e) + W_p + W_f\right] = 0.5\left[(1+1)\left(2{,}643\frac{\text{lb}}{\text{ft}}\right) + 662\frac{\text{lb}}{\text{ft}} + 600\frac{\text{lb}}{\text{ft}}\right] = 3{,}274\frac{\text{lb}}{\text{ft}}$$

Steel cylinder area:

$$A_y = \pi(D_y - t_y)(t_y) = \pi(44.5\ \text{in.} - 0.1046\ \text{in.})(0.1046\ \text{in.}) = 14.59\ \text{in.}^2$$

Transformed pipe wall area:

$$A_t = A_c + nA_y = 643\ \text{in.}^2 + 8.295(14.59\ \text{in.}^2) = 764\ \text{in.}^2$$

Steel cylinder moment of inertia:

$$I_s = \frac{\pi}{64}\left[D_y^{\ 4} - (D_y - 2t_y)^4\right] = \frac{\pi}{64}\left\{(44.5\ \text{in.})^4 - [44.5\ \text{in.} - 2(0.1046\ \text{in.})]^4\right\} = 3{,}594\ \text{in.}^4$$

Effective transformed (to concrete) pipe moment of inertia, including softening factor:

$$I_{eff} = \left[\frac{\pi}{64}(D_o^4 - ID^4) - I_s\right]\psi + nI_s$$

$$= \left[\frac{\pi}{64}\left[(51\ \text{in.})^4 - (42\ \text{in.})^4\right] - 3{,}594\ \text{in.}^4\right](0.2) + (8.295)(3{,}594\ \text{in.}^4) = 64{,}962\ \text{in.}^4$$

Characteristic beam on elastic foundation length (Eq. 9-10 from AWWA Manual M9):

$$\lambda = \sqrt[4]{\frac{k}{4E_c I_{eff}}} = \sqrt[4]{\frac{1{,}900 \text{ lb/in./in.}}{4\left(3{,}616{,}500 \text{ psi}\right)\left(64{,}962 \text{ in.}^4\right)}} = 0.006705 \frac{1}{\text{in.}}$$

As shown by Zarghamee et al. (2004), equations governing the behavior of finite length and semi-infinite length beams on elastic foundations can be derived from those for infinite beams on elastic foundations. For a finite length beam on elastic foundation, consider an infinite beam on elastic foundation and apply an unknown force, P_a , and an unknown moment, M_a , at point a, with x = 0 and unknown force, P_b , and moment, M_b , at point b, with x = L, where points a and b represent the ends of the pipe segment (Figure A-2). For a semi-infinite beam on elastic foundation, apply an unknown force, P_a , and an unknown moment, M_a , at point a, with x = 0 (Figure A-3). For given boundary conditions on displacement y, moment M, and shear force Q, at points a and b of the finite length beam and at point a of the semifinite beam, the values of the unknown forces and moments (P_a, M_a, Pb, M_b) are calculated by simultaneously solving equilibrium and compatibility equations based on the following equations for displacement, slope, shear, and moment at x = 0 and x = L on the beam of finite length and at point a with x = 0 of the beam of semi-infinite length.

For finite length beams on elastic foundation, the equations for y, θ, M, and Q are:

$$y\left(x,L,P_a,P_b,M_a,M_b\right) = \frac{P_a\lambda}{2k}A\left(\lambda,x\right) + \frac{P_b\lambda}{2k}A\left(\lambda,L-x\right) + \frac{M_a\lambda^2}{k}B\left(\lambda,x\right) + \frac{M_b\lambda^2}{k}B\left(\lambda,L-x\right)$$

Figure A-2 – Infinite Beam on Elastic Foundation with Applied Forces and Moments at the Ends of a Beam Segment

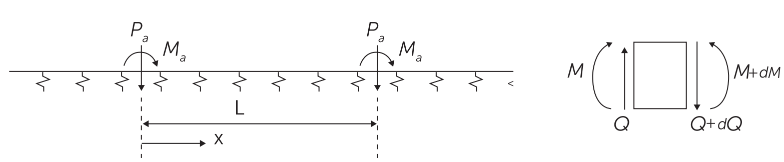
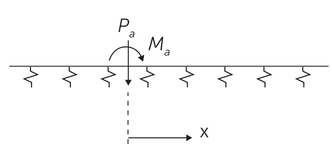


Figure A-3 – Infinite Beam on Elastic Foundation with Applied Force and Moment at the End of a Semi-Infinite Beam



$$\theta(x,L,P_a,P_b,M_a,M_b) = \frac{P_a\lambda^2}{k}B(\lambda,x) + = -\frac{P_a\lambda^2}{k}B(\lambda,x) + \frac{P_b\lambda^2}{k}B(\lambda,L-x) + \frac{M_a\lambda^3}{k}C(\lambda,x) - \frac{M_b\lambda^3}{k}C(\lambda,x)$$

$$M(x,L,P_a,P_b,M_a,M_b) = \frac{P_a}{4\lambda}C(\lambda,x) + \frac{P_b}{4\lambda}C(\lambda,L-x) + \frac{M_a}{2}D(\lambda,x) + \frac{M_b}{2}D(\lambda,L-x)$$

$$Q(x,L,P_a,P_b,M_a,M_b) = -\frac{P_a}{2}D(\lambda,x) + \frac{P_b}{2}D(\lambda,L-x) - \frac{M_a\lambda}{2}A(\lambda,x) + \frac{M_b\lambda}{2}A(\lambda,L-x)$$

For semi-infinite length beams on elastic foundation, the equations for y, θ, M, and Q are:

$$y(x,P_a,M_a) = \frac{P_a\lambda}{2k}A(\lambda,x) + \frac{M_a\lambda^2}{k}B(\lambda,x)$$

$$\theta(x,P_a,M_a) = -\frac{P_a\lambda^2}{k}B(\lambda,x) + \frac{M_a\lambda^3}{k}C(\lambda,x)$$

$$M(x,P_a,M_a) = \frac{P_a}{4\lambda}C(\lambda,x) + \frac{M_a}{2}D(\lambda,x)$$

$$Q(x,P_a,M_a) = -\frac{P_a}{2}D(\lambda,x) - \frac{M_a\lambda}{2}A(\lambda,x)$$

Where:

$$\lambda = \sqrt[4]{\frac{k}{4EI}}, k = \text{soil stiffness}, E = \text{modulus of elasticity}, I = \text{moment of intertia, and}$$

$$A(\lambda,x) = e^{-\lambda x}\left[\cos(\lambda x) + \sin(\lambda x)\right]$$
$$B(\lambda,x) = e^{-\lambda x}\sin(\lambda x)$$
$$C(\lambda,x) = e^{-\lambda x}\left[\cos(\lambda x) - \sin(\lambda x)\right]$$
$$D(\lambda,x) = e^{-\lambda x}\cos(\lambda x)$$

Let us assume that each leg of a symmetric bend with mechanically restrained joints consists of two finite and one semi-infinite beams on elastic foundation (BOEF) as shown in Figure A-4. Finite-element analysis of a thrust restraint of simple bends of pipelines with mechanically restrained joints has shown that joint rotation is primarily limited to the first three joints. Where more than three joints experience joint rotation, use of three-joint rotation is an approximation that is most likely conservative. When the accuracy demands, a larger number of beam segments may be considered.

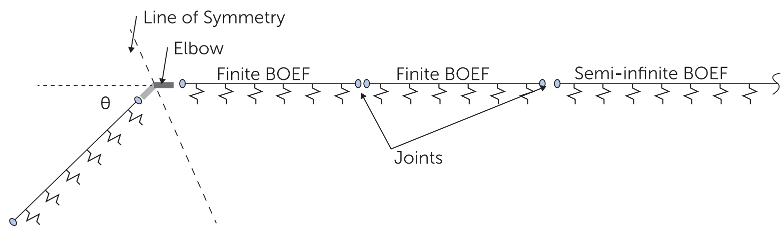
Let us assume that the number of finite-length pipe segments that act in restraining axial force is n_p, and for the first iteration, n_p is selected to be 2. The restrained length of pipe = $n_p L_P$, where L_P = the length of each finite-length pipe segment. The axial force at the first joint equals the number of finite-length pipe segments times the frictional resistance of the pipe. Assuming that two joints are fully rotated and thus have one-half of the slack, the axial motion is expressed by:

$$\delta_a = n_p\Omega + \frac{f_\mu \left(n_p L_P\right)^2}{2E_c A_t} = 2\left(\frac{1}{16}\text{ in.}\right) + \frac{\left(273\frac{\text{lb}}{\text{in.}}\right)\left[(2)(240\text{ in.})\right]^2}{2(3{,}616{,}500\text{ psi})(764\text{ in.}^2)} = 0.1364\text{ in.}$$

The transverse motion of the pipe:

$$\delta_b = \frac{\delta_a}{\tan\frac{\Delta}{2}} = \frac{0.1364\text{ in.}}{\tan\frac{75^\circ}{2}} = 0.1778\text{ in.}$$

Figure A-4 – Schematic of Mechanically Harnessed Joint with Joint Rotation



The total motion of the pipe:

$$\delta = \sqrt{\delta_a^2 + \delta_b^2} = \sqrt{(0.1364\text{ in.})^2 + (0.1778\text{ in.})^2} = 0.2241\text{ in.}$$

The axial thrust at each pipe joint:

At the first joint:

$$F_{jt1} = f_\mu n_p L = \left(273\ \frac{\text{lb}}{\text{in.}}\right)(2)(240\text{ in.}) = 131\text{ kip}$$

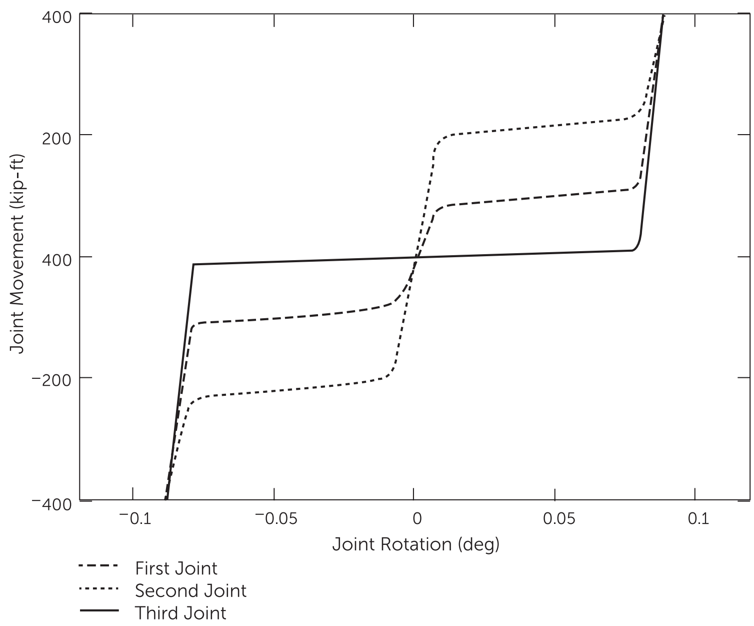
At the second joint:

$$F_{jt2} = f_\mu\left(n_p - 1\right)L = \left(273\ \frac{\text{lb}}{\text{in.}}\right)(2-1)(240\text{ in.}) = 65.6\text{ kip}$$

At the third joint, no axial tension remains.

For each joint, the moment rotation relationship can be calculated based on the axial force. The axial force alone can open the joint fully to a maximum of Ω. The moment alone can cause a joint rotation of $\theta_b = \Omega/D_j$. For combinations of axial force and moment that produce rotations less than θ_b, the moment is based on a single point of contact, i.e., $M = FD_j / 2$. For rotations greater than θ_b, the moments are calculated from the joint ring stiffness. The calculated moment-rotation curve for the first three joints is shown in Figure A-5. The moment-rotation relationship described by these curves is referred to as $M_j(\theta)$, indicating the moment at joint j is a function of the given angle contained within the parentheses.

Figure A-5 – Moment-Rotation Diagram



To determine the bending moment and shear in the pipe, a system of ten equations and ten unknowns is established. The unknowns are the four infinite BOEF forces and moments for the first beam segment (P_{a1}, P_{b1}, M_{a1}, M_{b1}), four for the second beam segment (P_{a2}, P_{b2}, M_{a2}, M_{b2}), and two for the semi-infinite beam (P_{a3}, M_{a3}). The calculation assumes a value for the movement of the bend δ, from which δ_a and δ_b are computed.

Equations of equilibrium and compatibility are now used to determine the pipeline response. These equations enforce force, moment, moment-rotation, and transverse motion compatibility.

	Evaluated at x = 0	Evaluated at x = L
$A(x) = e^{-\lambda x}\left[\cos(\lambda x) + \sin(\lambda x)\right]$	1.0	0.191
$B(x) = e^{-\lambda x}\sin(\lambda x)$	0.0	0.199
$C(x) = e^{-\lambda x}\left[\cos(\lambda x) - \sin(\lambda x)\right]$	1.0	-0.208
$D(x) = e^{-\lambda x}\cos(\lambda x)$	1.0	-0.008

1) Transverse deflection of first joint:

$$y(x, L_1, P_{a1}, P_{b1}, M_{a1}, M_{b1}) = \delta_b$$

$$\delta = y(x, L, P_{a1}, P_{b1}, M_{a1}, M_{b1}) = \frac{P_{a1}\lambda}{2k}A(x) + \frac{P_{b1}\lambda}{2k}A(L-x) + \frac{M_{a1}\lambda^2}{k}B(x) + \frac{M_{b1}\lambda^2}{k}B(L-x)$$

evaluated at x = 0.

$$0.178\text{ in.} = \frac{P_{a1}(0.0067)}{2(1,900\text{ lb/in./in.})}(1.0) + \frac{P_{b1}(0.0067)}{2(1,900\text{ lb/in./in.})}(0.191) + \frac{M_{b1}(0.0067^2)}{1,900\text{ lb/in./in.}}(0.199)$$

2) Moment rotation relationship at first joint:

$$M(0, L_1, P_{a1}, P_{b1}, M_{a1}, M_{b1}) = Mj\left[0 - \theta(0, L_1, P_{a1}, P_{b1}, M_{a1}, M_{b1})\right]$$

$$\frac{P_{a1}}{4\lambda}C(x) + \frac{P_{b1}}{4\lambda}C(L-x) + \frac{M_{a1}}{2}D(x) + \frac{M_{b1}}{2}D(L-x)$$

$$Mj\left(0 - \left[\frac{P_{a1}\lambda^2}{k}B(x) + \frac{P_{b1}\lambda^2}{k}B(L-x) + \frac{M_{a1}\lambda^3}{k}C(x) - \frac{M_{b1}\lambda^3}{k}C(L-x)\right]\right)$$

evaluated at x = 0.

$$\frac{P_{a1}}{4(0.0067)}(1.0) + \frac{P_{b1}}{4(0.0067)}(-0.208) + \frac{M_{a1}}{2}(1.0) + \frac{M_{b1}}{2}(-0.008)$$

$$= Mj\left[0 - \left(\frac{P_{b1}(0.0067^2)}{1,900\text{ lb/in./in.}}(0.199) + \frac{M_{a1}(0.0067^3)}{1,900\text{ lb/in./in.}}(1.0) - \frac{M_{b1}(0.0067^3)}{1,900\text{ lb/in./in.}}(-0.208)\right)\right]$$

3) Deflection compatibility at the second joint:

$$y\left(L_1,L_1,P_{a1},P_{b1},M_{a1},M_{b1}\right)=y\left(0,L_2,P_{a2},P_{b2},M_{a2},M_{b2}\right)$$

$$\frac{P_{a1}\lambda}{2k}A\left(x_1\right)+\frac{P_{b1}\lambda}{2k}A\left(L-x_1\right)+\frac{M_{a1}\lambda^2}{k}B\left(x_1\right)+\frac{M_{b1}\lambda^2}{k}B\left(L-x_1\right)$$

$$=\frac{P_{a2}\lambda}{2k}A\left(x_2\right)+\frac{P_{b2}\lambda}{2k}A\left(L-x_2\right)+\frac{M_{a2}\lambda^2}{k}B\left(x_2\right)+\frac{M_{b2}\lambda^2}{k}B\left(L-x_2\right)$$

evaluated at x_1 = L, x_2 = 0.

$$\frac{P_{a1}\left(0.0067\right)}{2\left(1,900\text{ lb/in./in.}\right)}\left(0.191\right)+\frac{P_{b1}\left(0.0067\right)}{2\left(1,900\text{ lb/in./in.}\right)}\left(1.0\right)+\frac{M_{a1}\left(0.0067^2\right)}{1,900\text{ lb/in./in.}}\left(0.199\right)$$

$$=\frac{P_{a2}\left(0.0067\right)}{2\left(1,900\text{ lb/in./in.}\right)}\left(1.0\right)+\frac{P_{b2}\left(0.0067\right)}{2\left(1,900\text{ lb/in./in.}\right)}\left(0.191\right)+\frac{M_{b2}\left(0.0067^2\right)}{1,900\text{ lb/in./in.}}\left(0.199\right)$$

4) Moment compatibility at the second joint:

$$M\left(L_1,L_1,P_{a1},P_{b1},M_{a1},M_{b1}\right)=M\left(0,L_2,P_{a2},P_{b2},M_{a2},M_{b2}\right)$$

$$\frac{P_{a1}}{4\lambda}C\left(x_1\right)+\frac{P_{b1}}{4\lambda}C\left(L-x_1\right)+\frac{M_{a1}}{2}D\left(x_1\right)+\frac{M_{b1}}{2}D\left(L-x_1\right)$$

$$=\frac{P_{a2}}{4\lambda}C\left(x_2\right)+\frac{P_{b2}}{4\lambda}C\left(L-x_2\right)+\frac{M_{a2}}{2}D\left(x_2\right)+\frac{M_{b2}}{2}D\left(L-x_2\right)$$

evaluated at x_1 = L, x_2 = 0.

$$\frac{P_{a1}}{4\left(0.0067\right)}\left(-0.208\right)+\frac{P_{b1}}{4\left(0.0067\right)}\left(1.0\right)+\frac{M_{a1}}{2}\left(-0.008\right)+\frac{M_{b1}}{2}\left(1.0\right)$$

$$=\frac{P_{a2}}{4\left(0.0067\right)}\left(1.0\right)+\frac{P_{b2}}{4\left(0.0067\right)}\left(-0.208\right)+\frac{M_{a2}}{2}\left(1.0\right)+\frac{M_{b2}}{2}\left(-0.008\right)$$

5) Shear compatibility at the second joint:

$$Q\left(L_1,L_1,P_{a1},P_{b1},M_{a1},M_{b1}\right)=Q\left(0,L_2,P_{a2},P_{b2},M_{a2},M_{b2}\right)$$

$$-\frac{P_{a1}}{2}D\left(x_1\right)+\frac{P_{b1}}{2}D\left(L-x_1\right)-\frac{M_{a1}\lambda}{2}A\left(x_1\right)+\frac{M_{b1}\lambda}{2}A\left(L-x_1\right)$$

evaluated at x_1 = L, x_2 = 0.

$$-\frac{P_{a1}}{2}(-0.008)+\frac{P_{b1}}{2}(1.0)-\frac{M_{a1}(0.0067)}{2}(0.191)+\frac{M_{b1}(0.0067)}{2}(1.0)$$

$$=-\frac{P_{a2}}{2}(1.0)+\frac{P_{b2}}{2}(-0.008)-\frac{M_{a2}(0.0067)}{2}(1.0)+\frac{M_{b2}(0.0067)}{2}(0.191)$$

6) Moment-rotation relationship at the second joint:

$$M(L_1,L_1,P_{a1},P_{b1},M_{a1},M_{b1})$$

$$=Mj\left[\theta(L_1,L_1,P_{a1},P_{b1},M_{a1},M_{b1})-\theta(0,L_2,P_{a2},P_{b2},M_{a2},M_{b2})\right]$$

$$\frac{P_a}{4\lambda}C(x_1)+\frac{P_b}{4\lambda}C(L-x_1)+\frac{M_a}{2}D(x_1)+\frac{M_b}{2}D(L-x_1)$$

$$=M_j\left(\begin{array}{l}-\frac{P_{a1}\lambda^2}{k}B(x_1)+\frac{P_{b1}\lambda^2}{k}B(L-x_1)+\frac{M_{a1}\lambda^3}{k}C(x_1)-\frac{M_{b1}\lambda^3}{k}C(L-x_1)\\-\left[\frac{-P_{a2}\lambda^2}{k}B(x_2)+\frac{P_{b2}\lambda^2}{k}B(L-x_2)+\frac{M_{a2}\lambda^3}{k}C(x_2)-\frac{M_{b2}\lambda^3}{k}C(L-x_2)\right]\end{array}\right)$$

evaluated at $x_1 = L$, $x_2 = 0$.

$$\frac{P_a}{4(0.0067)}(-0.208)+\frac{P_b}{4(0.0067)}(1.0)+\frac{M_a}{2}(-0.008)+\frac{M_b}{2}(1.0)$$

$$=M_j\left(\begin{array}{l}\frac{-P_{a1}(0.0067)^2}{1{,}900\text{ lb/in./in.}}(0.199)+\frac{M_{a1}(0.0067)^3}{1{,}900\text{ lb/in./in.}}(-0.208)-\frac{M_{b1}(0.0067)^3}{1{,}900\text{ lb/in./in.}}(1.0)\\-\left[\frac{-P_{b2}(0.0067)^2}{1{,}900\text{ lb/in./in.}}(0.199)+\frac{M_{a2}(0.0067)^3}{1{,}900\text{ lb/in./in.}}(1.0)-\frac{M_{b2}(0.0067)^3}{1{,}900\text{ lb/in./in.}}(-0.208)\right]\end{array}\right)$$

7) Deflection compatibility at the third joint:

$$y(L_2,L_2,P_{a2},P_{b2},M_{a2},M_{b2})=y(0,P_{a3},M_{a3})$$

$$\frac{P_{a2}\lambda}{2k}A(x_2)+\frac{P_{b2}\lambda}{2k}A(L-x_2)+\frac{M_{a2}\lambda^2}{k}B(x_2)+\frac{M_{b2}\lambda^2}{k}B(L-x_2)=\frac{P_{a3}\lambda}{2k}A(x_3)+\frac{M_{a3}\lambda^2}{k}B(x_3)$$

evaluated at $x_2 = L$, $x_3 = 0$.

$$\frac{P_{a2}\left(0.0067\right)}{2\left(1{,}900\text{ lb/in./in.}\right)}\left(0.191\right)+\frac{P_{b2}\left(0.0067\right)}{2\left(1{,}900\text{ lb/in./in.}\right)}\left(1.0\right)+\frac{M_{a2}\left(0.0067\right)^{2}}{1{,}900\text{ lb/in./in.}}\left(0.199\right)=\frac{P_{a3}\left(0.0067\right)}{2\left(1{,}900\text{ lb/in./in.}\right)}\left(1.0\right)$$

8) Moment compatibility at the third joint:

$$M\left(L_2,L_2,P_{a2},P_{b2},M_{a2},M_{b2}\right)=M\left(0,P_{a3},M_{a3}\right)$$

$$\frac{P_{a2}}{4\lambda}C\left(x_2\right)+\frac{P_{b2}}{4\lambda}C\left(L-x_2\right)+\frac{M_{a2}}{2}D\left(x_2\right)+\frac{M_{b2}}{2}D\left(L-x_2\right)=\frac{P_{a3}}{4\lambda}C\left(x_3\right)+\frac{M_{a3}}{2}D\left(x_3\right)$$

evaluated at $x_2 = L$, $x_3 = 0$.

$$\frac{P_{a2}}{4\left(0.0067\right)}\left(-0.208\right)+\frac{P_{b2}}{4\left(0.0067\right)}\left(1.0\right)+\frac{M_{a2}}{2}\left(-0.008\right)+\frac{M_{b2}}{2}\left(1.0\right)=\frac{P_{a3}}{4\left(0.0067\right)}\left(1.0\right)+\frac{M_{a3}}{2}\left(1.0\right)$$

9) Shear compatibility at the third joint:

$$Q\left(L_2,L_2,P_{a2},P_{b2},M_{a2},M_{b2}\right)=Q\left(0,P_{a3},M_{a3}\right)$$

$$-\frac{P_{a2}}{2}D\left(x_2\right)+\frac{P_{b2}}{2}D\left(L-x_2\right)-\frac{M_{a2}\lambda}{2}A\left(x_2\right)+\frac{M_{b2}\lambda}{2}A\left(L-x_2\right)=-\frac{P_{a3}}{2}D\left(x_3\right)-\frac{M_{a3}\lambda}{2}A\left(x_3\right)$$

evaluated at $x_2 = L$, $x_3 = 0$.

$$-\frac{P_{a2}}{2}\left(-0.008\right)+\frac{P_{b2}}{2}\left(1.0\right)-\frac{M_{a2}\left(0.0067\right)}{2}\left(0.191\right)+\frac{M_{b2}\left(0.0067\right)}{2}\left(1.0\right)=-\frac{P_{a3}}{2}\left(1.0\right)-\frac{M_{a3}\left(0.0067\right)}{2}\left(1.0\right)$$

10) Moment-rotation relationship at the third joint:

$$M\left(L_2,L_2,P_{a2},P_{b2},M_{a2},M_{b2}\right)=M_j\left[\theta\left(L_2,L_2,P_{a2},P_{b2},M_{a2},M_{b2}\right)-\theta\left(0,P_{a3},M_{a3}\right)\right]$$

$$\frac{P_{a2}}{4\lambda}C\left(x_2\right)+\frac{P_{b2}}{4\lambda}C\left(L-x_2\right)+\frac{M_{a2}}{2}D\left(x_2\right)+\frac{M_{b2}}{2}D\left(L-x_2\right)$$

$$=M_j\left(\begin{array}{l}\dfrac{-P_{a2}\lambda^2}{k}B\left(x_2\right)+\dfrac{P_{b2}\lambda^2}{k}B\left(L-x_2\right)+\dfrac{M_{a2}\lambda^3}{k}C\left(x_2\right)-\dfrac{M_{b2}\lambda^3}{k}C\left(L-x_2\right)\\ -\left[\dfrac{-P_{a3}\lambda^2}{k}B\left(x_3\right)+\dfrac{M_{a3}\lambda^3}{k}C\left(x_3\right)\right]\end{array}\right)$$

evaluated at $x_2 = L$, $x_3 = 0$.

$$\frac{P_{a2}}{4\left(0.0067\right)}\left(-0.208\right)+\frac{P_{b2}}{4\left(0.0067\right)}\left(1.0\right)+\frac{M_{a2}}{2}\left(-0.008\right)+\frac{M_{b2}}{2}\left(1.0\right)$$

$$=M_j\left[-\frac{P_{a2}\left(0.0067\right)^{2}}{1{,}900\text{ lb/in./in.}}\left(0.199\right)+\frac{M_{a2}\left(0.0067\right)^{3}}{1{,}900\text{ lb/in./in.}}\left(-0.208\right)-\frac{M_{b2}\left(0.0067\right)^{3}}{1{,}900\text{ lb/in./in.}}\left(1.0\right)-\frac{M_{a3}\left(0.0067\right)^{3}}{1{,}900\text{ lb/in./in.}}\left(1.0\right)\right]$$

The solutions to these equations are:

M_{a1} = -211.6 kip-ft
M_{b1} = 24.7 kip-ft
M_{a2} = -179.7 kip-ft
M_{b2} = 22.3 kip-ft
M_{a3} = 33.9 kip-ft
P_{a1} = 100.3 kip
P_{b1} = -1.63 kip
P_{a2} = 10.38 kip
P_{b2} = -1.79 kip
P_{a3} = -5.57 kip

Back substitution shows that all ten equations have been satisfied. Using the BOEF equations, the displacement, moment, slope, and shear at any point on the pipe can be calculated. Figures A-6 to A-10 show the response of the pipeline model for a restraint length of two pipe segments. Note that for this example the rotation at the joints is small relative to a fully rotated harness joint of Ω/D_j = 0.08 degrees, with the first joint rotating about 0.02 degrees, and the second and third joints showing a much smaller relative rotation.

The joints are not fully rotated and therefore allow axial joint extension. The joint rotation, θ_j, is 4.01×10^{-4} radians at joint 1, 9.47×10^{-5} radians at joint 2, and 8.49×10^{-5} radians at joint 3. The axial joint extension available is: $\Omega_a = \Omega - (D_j/2)\theta_j$ = 0.053 in. at joint 1, 0.060 in. at joint 2, and 0.061 in. at joint 3. This calculation has assumed two pipe segments are effective in axial restraint in the calculation of axial force in the pipe and at the joints. To maintain axial compatibility, the axial deflection, δ_a, is limited by:

Figure A-6 – Displacement Along the Pipe Length

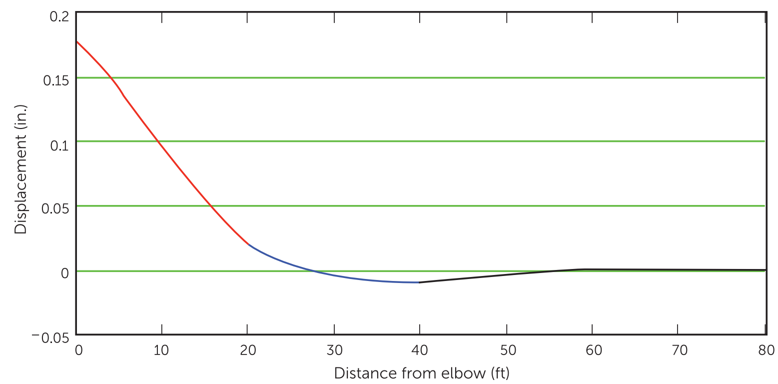


Figure A-7 – Moment Along the Pipe Length

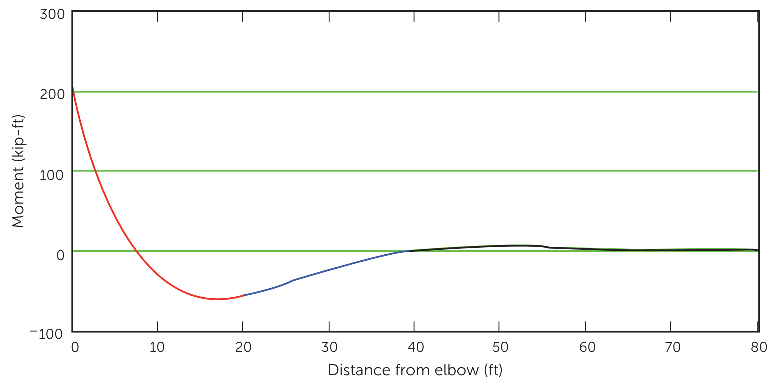


Figure A-8 – Rotation Along the Pipe Length

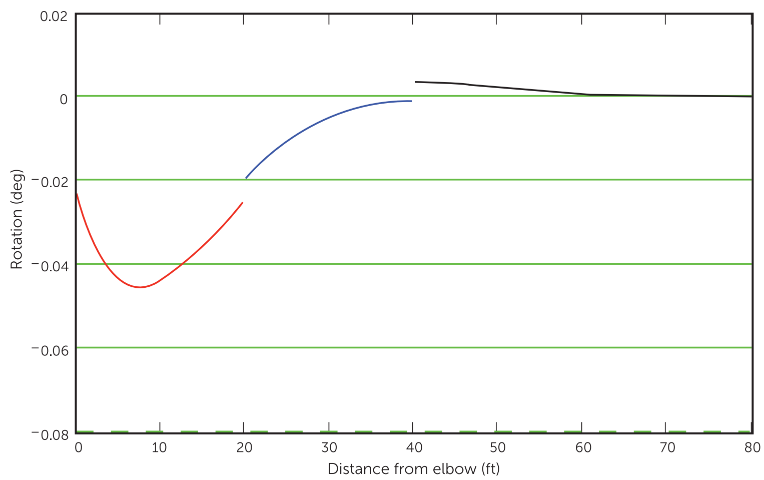


Figure A-9 – Shear Along the Pipe Length

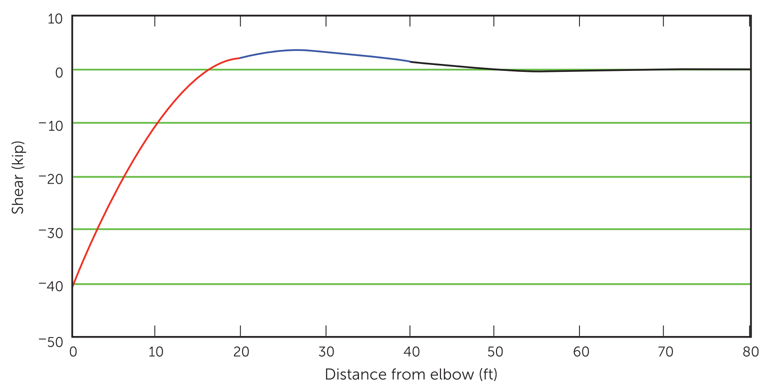
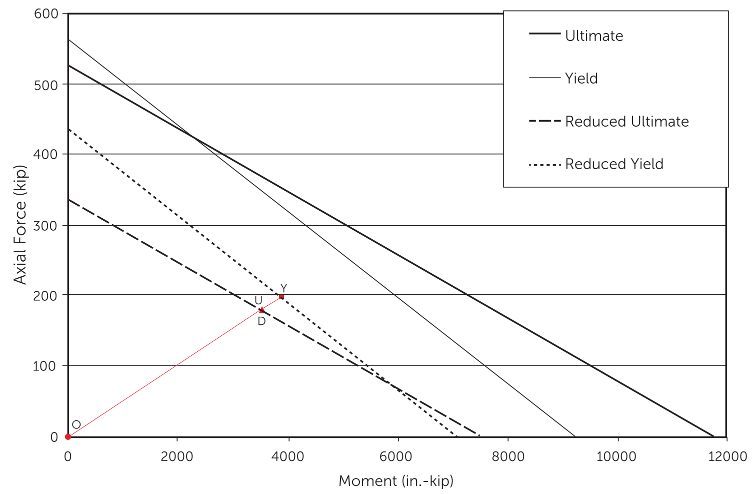


Figure A-10 – Interaction Diagram for Example 3



The axial force-moment interaction diagrams corresponding to the ultimate strength and the onset of yielding of the steel cylinder are both non-linear, but have been approximated by a linear relationship here for ease of computation of the examples.

$$\Omega_{a1}+\Omega_{a2}+\frac{f_{\mu}\left(2\times L_{P}\right)^{2}}{2E_{c}A_{t}}\leq\delta_{a}\leq\Omega_{a1}+\Omega_{a2}+\Omega_{a3}+\frac{f_{\mu}\left(2\times L_{P}\right)^{2}}{2E_{c}A_{t}}$$

$$(0.053+0.060)\text{ in.}+\frac{\left(273\frac{\text{lb}}{\text{in.}}\right)\left(2\times 240\text{ in.}\right)^{2}}{2\left(3{,}616{,}500\text{ psi}\right)\left(764\text{ in.}^{2}\right)}\leq 0.1364\text{ in.}$$

$$\leq(0.053+0.060+0.061)\text{ in.}+\frac{\left(273\frac{\text{lb}}{\text{in.}}\right)\left(2\times 240\text{ in.}\right)^{2}}{2\left(3{,}616{,}500\text{psi}\right)\left(764\text{ in.}^{2}\right)}$$

$$0.1244\leq 0.1364\leq 0.1854=\text{True}$$

Axial compatibility has been satisfied. If it had not been satisfied, further iteration would be required.

5. For n_p =2, the moment at the first joint, M = 2,480 kip-in.; the axial force at the first joint (calculated above), F = 131 kip; the shear at the first joint, Q = -41.4 kip; and the thrust required to produce the motion is:

$$T = -2Q\cos\frac{\Delta}{2} + 2F\sin\frac{\Delta}{2} + 2k\delta l_b\cos\frac{\Delta}{2}$$

$$= -2(-41.4\text{ kip})\left(\cos\frac{75°}{2}\right) + 2(131\text{ kip})\left(\sin\frac{75°}{2}\right)$$

$$+2(1{,}900\text{ kip/in./in.})(0.224\text{ in.})(38.64\text{ in.})\left(\cos\frac{75°}{2}\right) = 251.3\text{ kip}$$

The pressure required to produce this thrust, from (Eq. 9-1E from AWWA Manual M9):

$$P = \frac{T}{2A\sin\frac{\Delta}{2}} = \frac{251.3\text{ kip}}{2(1{,}590\text{ in.})\left(\sin\frac{75°}{2}\right)} = 129.8\text{ psi}$$

Because this pressure is less than P_{ftweff} = 1.25P_{weff} = 180 psi, the process is repeated for three pipe lengths of axial friction, n_p = 3. The resulting moment at the first joint, M = 3,911 in.-kip; the axial force at the first joint, F = 197 kip; the shear at the first joint, Q = −65.3 kip, the thrust required to produce the motion is 384.0 kip, and the pressure required to produce the thrust is 198.3 psi.

Linear interpolation between the results, including the at-rest state, is used to determine the moment, axial force, and axial force dissipation length, at the desired pressure. For a design pressure of 180 psi, the interpolated results are:

M = 3,528 in.-kip
F = 179.2 kip

Thrust dissipation length required:

$$L_{ft} = \frac{F}{f_\mu} = \frac{179.2\text{ kip}}{3.27\text{ kip/ft}} = 55\text{ ft}$$

This length is between 2 and 3 restrained pipe lengths, and is therefore consistent with the above analysis.

6. These results are then compared with the moment interaction diagram to determine if the result is acceptable. The interaction diagram is constructed as described in Design Example 2. As can be seen on the interaction diagram, Figure A-10, this design meets the requirements.

The margin of safety (MS) is the ratio of the pipe strength to the required pipe strength including all factors of safety. Therefore, it is a measure of how much the design exceeds the required factors of safety. The margin of safety against ultimate criterion is:

$$MS_{ultimate} = \frac{1}{\frac{M}{M_{ultimate}} + \frac{F}{F_{ultimate}}}\left(\frac{1}{1.56}\right) = \frac{1}{\frac{3{,}528 \text{ in.-kip}}{11{,}673 \text{ in.-kip}} + \frac{179.2 \text{ kip}}{525 \text{ kip}}}\left(\frac{1}{1.56}\right) = 1.00$$

The MS against the yield criterion is:

$$MS_{yield} = \frac{1}{\frac{M}{M_{yield}} + \frac{F}{F_{yield}}}\left(\frac{1}{1.3}\right) = \frac{1}{\frac{3{,}528 \text{ in.-kip}}{9{,}221 \text{ in.-kip}} + \frac{179.2 \text{ kip}}{565 \text{ kip}}}\left(\frac{1}{1.3}\right) = 1.10$$

Therefore, the minimum cylinder thickness required is:

$$t_{yreq} = \frac{0.1046 \text{ in.}}{1.0} = 0.1046 \text{ in.}$$

If the lower of the two MS values had been greater than 1.00, the minimum cylinder thickness required would be less than 0.1046 in. (0.1046 ÷ MS).

The cylinder at the first joint is selected to be 12 ga = 0.1046 in.

7. Determine the required cylinder thicknesses over the thrust dissipation length:

The pipe axial force, and therefore required cylinder thickness, diminishes on an approximately straight-line basis to zero at the thrust dissipation length from the bend, and the moment diminishes more rapidly. Straight line interpolation is therefore used to determine the required length of each cylinder thickness.

$$L_{12\ ga} = L_{ft} - \frac{t_{y\ 14ga}}{t_{yreq}} L_{ft} = 55 \text{ ft} - \frac{0.0747 \text{ in.}}{0.1046 \text{ in.}}(55 \text{ ft}) = 16 \text{ ft}$$

$$L_{14\ ga} = L_{ft} - L_{12\ ga} - \frac{t_{y\ 16ga}}{t_{yreq}} L_{ft} = 55 \text{ ft} - 16 \text{ ft} - \frac{0.0598 \text{ in.}}{0.1046 \text{ in.}}(55 \text{ ft}) = 8 \text{ ft}$$

$$L_{16\ ga} = L_{ft} - L_{12\ ga} - L_{14\ ga} = 55 \text{ ft} - 16 \text{ ft} - 8 \text{ ft} = 31 \text{ ft}$$

See Figure A-13 for the minimum and actual restrained footages and cylinder thicknesses based on the use of 20 ft lengths. For this example, only 16 ga and 12 ga cylinder thicknesses were used due to the very short footage of 14 ga required and the convenience of using only full lengths of pipe.

The required restrained joint strength (Fj) to resist material failure or joint leakage is calculated from Eq. 9-15 from AWWA Manual M9:

$$F_j = \pi\left(D_y - t_{yreq}\right) t_{yreq} f_y$$

$$= \pi\left(44.5 - 0.1046\right)\left(0.1046\right)\left(36{,}000 \text{ psi}\right)$$

$$= 525{,}200 \text{ lbs}$$

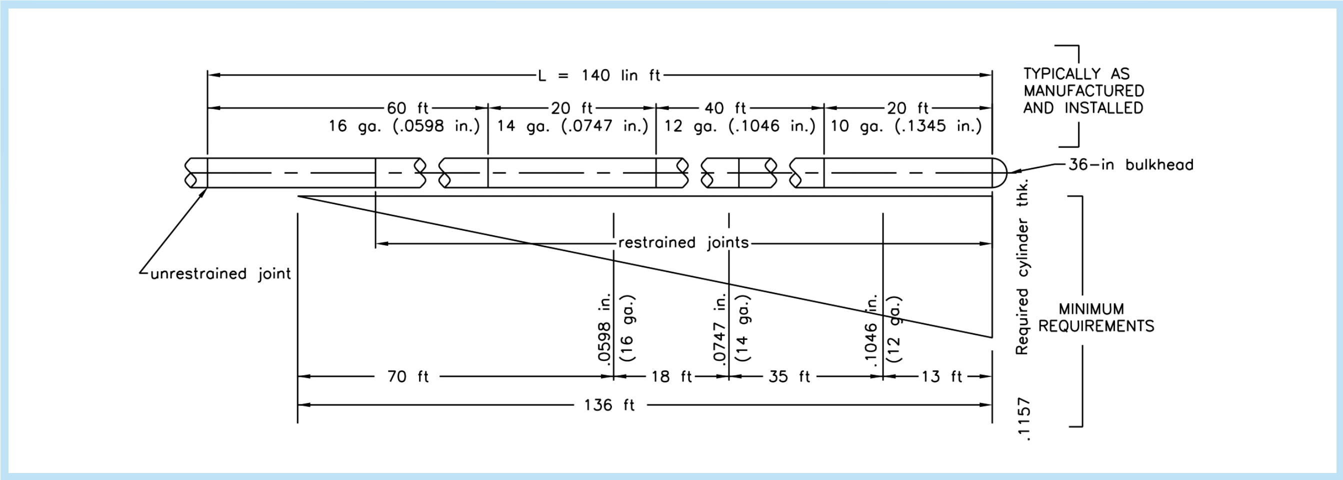
The methodology used for the previous example can be adapted for a variety of conditions, including bends with a greater number of finite-length restrained pipe, bends restrained by tied pipe using a mix of welded and mechanically restrained joints, bends with restrained pipe of different stiffnesses, bends with steel-cylinder extensions which exceed the normal length limits, and the use of mechanically restrained joints installed fully closed, partially extended, or fully extended.

Table A-1 – Summary of Design Examples and Results of Detailed Calculation

Conditions and Parameters	Example 1	Example 2	Example 3
Bend Conditions:			
Fitting	Bulkhead	45° Bend	75° Bend
Joint type	Any	Welded	Mech. rest.
Bend lenth, l_b, in.	—	24.6	38.64
Pipe Properties:			
Pipe type	LCP (C301)	BWP (C303)	ECP (301)
Nominal pipe inside diameter, ID, in.	36	54	42
Pipe weight, W_p,* lb/ft	404	518	662
Core outside diameter, OD, in.	40.5	55.875	49.0
Pipe outside diameter, D_o, in.	42.50	58.38	51.00
Core thickness, h_c, in.	2.25	0.9375	3.5
Mortar coating thickness, t_m, in.	1.0	1.25	1.0
Effective core thickness, h_c, in.	2.25	2.188	3.5
Joint diameter, D_j, in.	41.0	56.375	45.0
Cylinder outside diameter, D_y, in.	40.5	55.875	44.5
Minimum cylinder thickness	16 ga	$^3/_{16}$ in.	16 ga
Pipe laying length, L_p, ft	20 ft	40 ft	20 ft
Pressure Conditions:			
Working pressure, P_w, psi	150	200	140
Transient pressure, P_t, psi	80	100	60
Field test pressure, P_{ft}, psi	200	250	180
Effective working pressure, P_{weff}, psi	164	214	144
Soil Conditions:			
Cover depth, H, ft	4	6	5
Soil type	III	II	III
Soil weight, γ_s, pcf	114	120	114
Soil stiffness, k, psi	1,900	3,400	1,900
Pipe-to-soil friction coefficient, μ	0.5	0.5	0.5
Angle of internal friction, ϕ, in degrees	30	34	30
Material Properties:			
Concrete strength, f'_c, psi	6,000	4,500	4,500
Modulus of elasticity for concrete, E_c, psi	3,942,500	3,616,500	3,616,500
Concrete tensile strength, f_t, psi	387	335	335
Concrete weight, y_c, pcf	145	145	145
Steel cylinder yield strength, f_y, psi	36,000	36,000	36,000
Modulus of elasticity for steel, E_y, psi	30,000,000	30,000,000	30,000,000
Modular ratio, n	7,609	8.295	8.295
Steel yield strain, ε_y	0.0012	0.012	0.0012
Concrete strength at steel yield strain, f_{cy}, psi	123	82	82
Results:			
Selected cylinder thickness, t_y	10 ga	0.1875 in.	12 ga
Ultimate moment strength, $M_{ultimate}$, in.-kip	—	27,921	11,673
Ultimate tensile strength, $F_{ultimate}$, kip	—	1,181	525
Yield moment strength, M_{yield}, in.-kip	—	19,611	9,221
Yield tensile strength, F_{yield}, kip	—	1,199	565
Pipe moment at first joint, M, in.-kip	—	7,392	3,528
Pipe axial force at first joint, F, kip	—	373	179
Thrust dissipation length, L_{ft}, ft	136	82	55
Margin of safety, ultimate	—	1.10	1.00
Margin of safety, yield	—	1.12	1.10

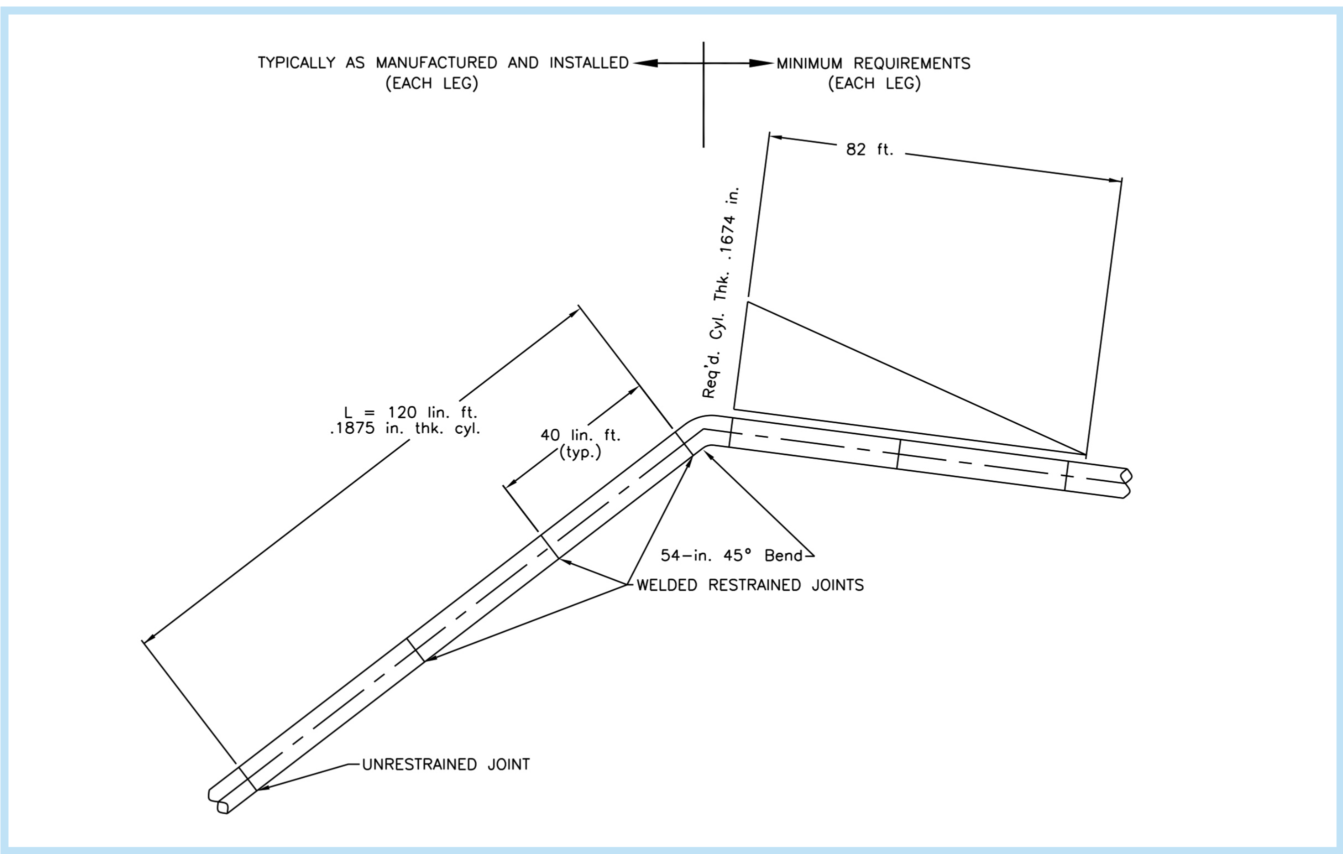
* Pipe weight for ANSI/AWWA C300, C301 and C302 is calculated from page 69, Step 3 in AWWA Manual M9. Pipe weight for ANSI/AWWA C303 pipe is obtained from the manufacturer.

Figure A-11



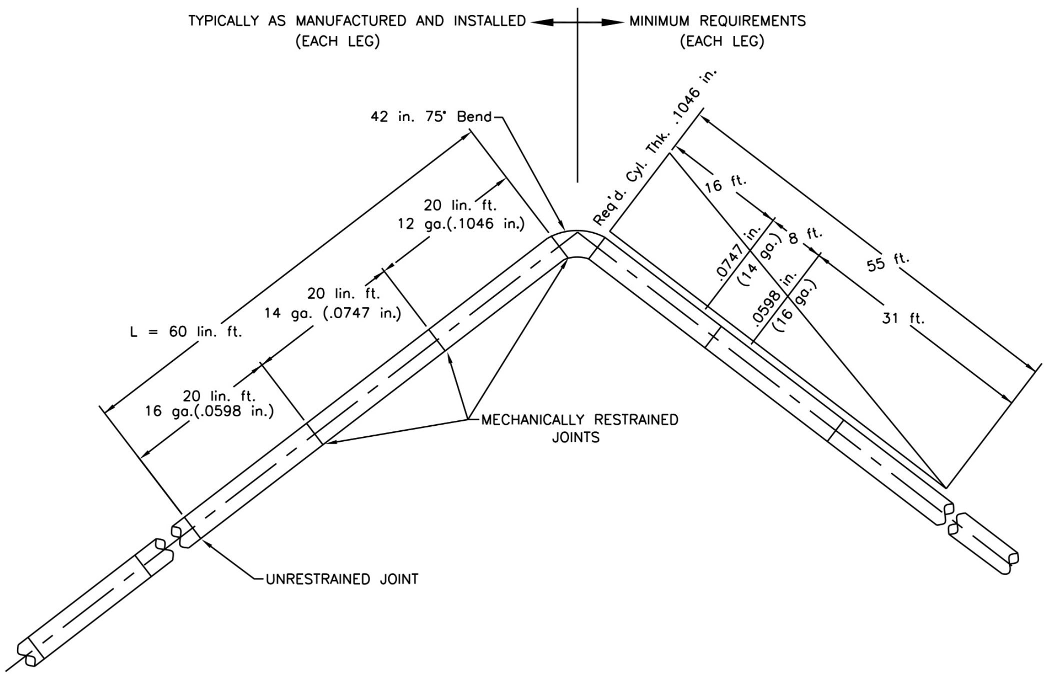
Thrust restraint requirements for 36 in. C301 (LCP) bulkhead with restrained joints at P_w = 150 psi, P_t = 80 psi, and P_{ft} = 200 psi with 4-ft earth cover in Type III soil.

Figure A-12



Thrust restraint requirements for 54 in. C303 45° bend with welded restrained joints at P_w = 200 psi, P_t = 100 psi, and P_{ft} = 250 psi with 6-ft earth cover in Type II soil.

Figure A-13



Thrust restraint requirements for 42 in. C301 (ECP) bend with mechanically restrained joints at P_w = 140 psi, P_t = 60 psi, and P_{ft} = 180 psi with 5-ft earth cover in Type III soil.

Table A-2 – Summary of Additional Design Examples

Conditions and Parameters	Example 4	Example 5	Example 6	Example 7	Example 8
Bend Conditions:					
Fitting	22.5° Bend	90° Bend	30° Bend	22.5° Bend	90° Bend
Joint type	Mech. Restr.	Mech. Restr.	Welded	Welded	Mech. Restr.
Bend length, l_b, in.	15.6	34.8	27.8	19.2	98.4
Pipe Properties:					
Pipe type	ECP (C301)	BWP (C303)	ECP (C301)	(C300)	ECP (C301)
Nominal pipe inside diameter, ID, in.	72	30	96	84	96
Pipe weight, W_p*, lb/ft	1,614	245	2,456	2,328	2,456
Core outside diameter, OD, in.	83	31.88	109	100	109
Pipe outside diameter, D_o, in.	85.00	34.13	111.00	100.00	111.00
Core thickness, h_c, in.	5.5	0.9375	6.5	8.0	6.5
Mortar coating thickness, t_m, in.	1	1.125	1	NA	1
Joint diameter, D_j, in.	76.375	32.375	101.125	88.75	101.125
Cylinder outside diameter, D_y, in.	75.5	31.875	100.25	87.875	100.25
Minimum cylinder thickness	16 ga	10 ga	16 ga	12 ga	16 ga
Pipe laying length, L_p, ft	20	40	20	20	20
Pressure Conditions:					
Working pressure, P_w, psi	125	250	175	72	150
Transient pressure, P_t, psi	50	0	85	0	75
Field test pressure, P_{ft}, psi	150	250	225	72	180
Effective working pressure, P_{weff}, psi	125	250	185.7	72	160.7
Soil Conditions:					
Cover depth, H, ft	4	3	4	5	4
Soil type	IV	II	V	III	III
Soil weight, γ_s, pcf	112	120	110	114	114
Soil stiffness, k, psi	1,100	3,400	425	1,900	1,900
Pipe-to-soil friction coefficient, μ	0.4	0.5	0.3	0.5	0.5
Angle of internal friction, ϕ, degrees	20	34	20	30	30
Material Properties:					
Concrete strength, f'_c, psi	4,500	4,500	4,500	4,500	4,500
Modulus of elasticity for concrete, E_c, psi	3,616,500	3,616,500	3,616,500	3,616,500	3,616,500
Concrete tensile strength, f_t, psi	335	335	335	335	335
Concrete weight, γ_c, pcf	145	145	145	145	145
Steel cylinder yield strength, f_y, psi	36,000	36,000	36,000	36,000	36,000
Modulus of elasticity for steel, E_y, psi	30,000,000	30,000,000	30,000,000	30,000,000	30,000,000
Modular ratio, n	8.295	8.295	8.295	8.295	8.295
Steel yield strain, ε_y	0.0012	0.0012	0.0012	0.0012	0.0012
Concrete strength at steel yield strain, f_{cy}, psi	82.6	82.6	82.6	82.6	82.6
Results:					
Selected cylinder thickness, t_y	12 ga	10 ga	0.3125 in.	12 ga	0.3125 in.
Ultimate moment strength, $M_{ultimate}$, in.-kip	34,210	6,750	166,348	47,220	166,348
Ultimate tensile strength, $F_{ultimate}$, kip	892	483	3,532	1,038	3,532
Yield moment strength, M_{yield}, in.-kip	29,301	4,736	121,544	43,072	121,544
Yield tensile strength, F_{yield}, kip	990	492	3,679	1,186	3,679
Pipe moment at first joint, M, in.-kip	11,473	2,128	34,200	4,147	50,953
Pipe axial force at first joint, F, kip	133	150	1,293	381	1,146
Thrust dissipation length, L_{ft}, ft	30	107	274	49	145
Margin of safety, ultimate	1.32	1.02	1.12	1.41	1.02
Margin of safety, yield	1.46	1.02	1.22	1.84	1.05

* Pipe weight for C300, C301 and C302 is calculated from page 69, Step 3 in AWWA Manual M9. Pipe weight for C303 pipe is obtained from the manufacturer.

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AMERICAN
Concrete Pressure Pipe
ASSOCIATION

ACPPA Headquarters
3900 University Drive
Suite 110
Fairfax, VA 22030-2513

T 703.273.7227
F 703.273.7230

www.acppa.org